The Quantum Metric*

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-
Fundamental Rest Mass Quanta as Simple Harmonic Oscillations of the Spacetime Continuum at Resonant Frequency and Wave Number, Driven by Cosmic Expansion

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Abstract

This analysis provides a physical, i.e. geometric, as well as mathematical, model of quantization, by way of a fundamental discrete oscillation/rotation, of a classical spacetime continuum that is a function of the exponential expansion of that spacetime. Quantum gravity arises naturally as the differential of that oscillatory transverse wave force with respect to expansion stress and the strong interaction as the operation of that wave force between two or more quanta within a shared force domain. This quantum state is expressed as a modification of a chargeless extreme Kerr metric with an oscillation of the ϕ coordinates imposed by continuity conditions which prevent coordinate entanglement. It thereby constitutes a physical spinor, constituting the quantum magnetic field and the property of ½ spin and isospin in the presence of other quanta. The ergosphere of this quantum metric is the domain of the strong interaction. Finally, it shows that from a universal bookkeeper reference frame, the fundamental quantum scale is the neutron scale, for which the Planck scale is the current differential.

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Introductory Assumptions

It is a fundamental assumption of this analysis that the 3-dimensional physical space of observation and experimentation is undergoing a phenomenological expansion at an exponential rate. Symmetry conditions require that what is stated as an expansion from any local reference frame must be expressed as a condensation or contraction of that locus away from its surrounding extents from the universal reference frame. From either conceptualization the ratio of universal extent to a local unit length, r_0, is increasing. With respect to the 4-dimensional spacetime of general relativity, such expansion means that the proportion between the local unit standards of space and time must remain constant during this expansion for the speed of light in vacuo to remain invariant.

The 3 dimensions of space constitute a 3-manifold without boundary which is embedded in a 4-manifold. Generalization from lower dimensions suggests that the 3-manifold is the boundary of a 4-core or 4-ball, currently in an apparent expansion phase, conceivably part of a universal harmonic oscillation. Of interest is that the 4-stress of expansion can be represented in 3-space as an isotropic 3-stress, as by analogy a 3-stress of an expanding 3-ball on its 2-sphere boundary, i.e. tension, can be represented as an isotropic 2-stress, i.e. a shear, in the boundary 2-sphere.

Spacetime in such model has a potential energy density that is converted to isotropic stress and kinetic strain with expansion. The decomposition of that stress into orthogonal alternations of tension, shear, and torsion stress results in a local strain oscillation within and of the 3-space. This oscillating strain, in turn, is registered as quantum spin energy and the rest of the various quantum spin functions. Quantization of spacetime and the physical properties of quanta are thus emergent properties of spacetime under isotropic expansion.

0 – Kinematics and the Geometrization of Time

> "Mechanics . . . is generally regarded as consisting of kinematics and dynamics. Kinematics . . . is the science that deals with the motions of bodies or particles without any regard to the causes of these motions. Studying the positions of bodies as a function of time, kinematics can be conceived as a space-time geometry of motions, the fundamental notions of which are the concepts of length and time. By contrast, dynamics, . . . is the science that studies the motions of bodies as the result of causative interactions. As it is the task of dynamics to explain the motions described by kinematics, dynamics requires concepts additional to those used in kinematics, for "to explain" goes beyond "to describe"."[1]

To take up the task set forth by Max Jammer, we might look for explanation of dynamics in a greater understanding of those "concepts additional", chief of which is mass; in particular we might also seek "to explain" those additional concepts through a more detailed description of the kinematic concepts of length and time. We would seek to find a definition of mass as a measure of length and/or time. In order to properly undertake such an investigation, we must first examine the concepts of length and time.

Length is a concept used to quantify the phenomenological fact of the spatial separation of entities, where entity might be any distinction within the field of observation, including the two ends of a ruler. Defining each and every element of a language in terms of that language necessarily involves some degree of circuitous reference. Whether recognized as such or not, certain primary concepts must be employed in a linguistic development, which are understood apart from the language itself, based on the assumption of a common experience by those employing that language. The quality of spatial separation is such a concept. From birth the vast majority of humans understand proximity to and separation from the warmth and sustenance of a mother, long before they understand speech, and they begin to conceptualize and quantify that separation at least as soon as they are able to visually focus and manually reach.

Two intelligent individuals who have never had a sense of sight or touch would have a difficult time developing, let alone communicating, a concept of length, though the concept of frequency and intensity would be accessible and communicable through the senses of hearing, smell, and taste. These same two sightless individuals might have little trouble developing a concept of time as a quantification of the phenomenological fact of the temporal separation of occurrences, such as the wait for the next feeding or the rate of their mother's heart beat while being held.

It is of interest that the magnitude of such time is commonly and customarily referred to as a length. We ask how <u>long</u> it has been since the last time we experienced some event, and we ask how <u>long</u> it is from point A to point B, generally meaning the duration of the trip rather than the distance. Perhaps most significantly, we crave or <u>long</u> for something

[1]Jammer, M. 2000, <u>Concepts of Mass in Contemporary Physics and Philosophy</u>, (Princeton: Princeton University Press) p. 5

we want which is remote from us in time or space, for something which we would like to have present. Humans appear to easily conflate measures of separation in time and in space with one term, length, and to join them together in a term for the rate with which some desired goal is approached, speed. However, there is no more than a conventional preference for the ordering of that relationship, as a mile in four minutes and a four minute mile indicate the same race speed or

$$c_{race} = \frac{\text{1 mile}}{\text{4 minutes}} = \frac{\text{4 minutes}}{\text{1 mile}} = \frac{\text{1 space or time interval}}{\text{1 time or space interval}}. \tag{0.1}$$

In a similar fashion, we can state a number of times per time or of lengths per length, i.e. a frequency in time or space, as

$$f_t = \frac{\text{4 flashes}}{\text{1 second}}, \text{ or } f_l = \frac{\text{3 feet}}{\text{1 yard}}. \tag{0.2}$$

A length of spatial or temporal separation can be termed an interval between entities or events, as is done in general relativity. Obviously a single entity can have multiple events, as with a flashing beacon, and we might suppose less obviously, that a single event can have multiple entities, as would be the case with a “big bang” or similar font of cosmic inception, as well as multiple perceptions of the event. To any well-thinking individual, this does not mean that temporal and spatial separation, or simply put, time and distance, are the same qualities by virtue of the use of this common reference term, but it does suggest that we might equate them mathematically with the use of some universally acknowledged gauge or standard of proportion, and we look for some extremum rate of change or motion as a basis for that gauge. Thus in relativity the speed of light, held to be a maximum, is used to gauge a length of time, converting it to a length of distance. We might also use as our gauge some rate held to be at or approaching some minimum, for example the expansion rate of space itself, the Hubble rate which on a local level is quite small, approximately 7.87×10^{-27} times smaller than the speed of light.

It bears emphasizing that the use of the same term for a separation by time and by space can be misleading. Spatial length is considered a primary concept in this discussion, understood by common experience (of the sight and touch gifted) and ultimately outside the capacity of formal, non-self-referent definition. In simplest manner, its magnitude is determined by holding two objects in proximity, one of which is a standard and the other of which is a test object. We might also consider temporal length as a primary concept, especially if we were one of our two sightless, non-tactile individuals, for whom it would necessarily be so. However, we tend to define time <u>quantitatively</u> in terms of a primary length concept, as with the inverse speed of light, as a comparison of the length rate of change along the circumference of a clock face contemporaneous with some other change.

Taking a hint from the nomenclature of simple arithmetic, we state that a velocity is the distance transited by some object divided by the number of <u>times</u> some other distance is transited at a regular rate, such as the number of times the end of a hand on a clock transits a distance on the circumference of the clock face designated as a unit standard interval. In the final analysis it is a comparison of two physical lengths, where the customary human standard itself is gauged to correspond with the tangential distance the earth rotates at the equator during (approximately) $1/24^{th}$, $1/1{,}440^{th}$, or $1/86{,}400^{th}$ of its diurnal cycle per hour, minute, or second respectively.

In similar fashion we can state that a frequency is the duration of some not-necessarily-spatial change, i.e. the flashing of a beacon, divided by the number of times that the end of the standard hand transits a standard interval or multiples or fractions thereof. Time is a comparison of two different rates of change, so that in the absence of motion or, more abstractly, change, time does not exist.

The reader may object that it is not the length transited by the end of the clock hand but rather its angular speed, the number of degrees, or radians, θ, transited, that marks out time. The circumferential length and angular speed are, of course, related by the length of the hand itself. All 60 second analog clocks move ideally at the same angular rate, regardless of the length of their hands, resulting in a varied velocity at hand tip that is a function of the hand length.

If we disregard real physical dynamics induced by the mass of the clock arm, we might envision that its velocity is limited at the end of its moving hand by the speed of light. Therefore, in an ideal clock we might stipulate that the length of time taken for light to travel from the center of the clock face to the end of the hand, be it hour, minute, second, nanosecond or yoctosecond, is equal to the length of time for the tip of the hand to travel the same distance tangentially about the face, i.e. for one radian. Thus its angular frequency, ω, will be inversely related and gauged to the length of its arm, r, or alternately by its angular wave length, $\lambda\!\!\!^{-}$, or its inverse, the angular wave number, κ, by some constant velocity, c, given by the familiar relationships

$$\omega = \frac{d\theta}{dt} = \frac{c}{r} = \frac{c}{\lambda\!\!\!^{-}} = c\kappa \,. \tag{0.3}$$

Some rearrangement and integration of the angular measure, using a normalized value for the speed, $c = 1$, gives

$$r\int_0^1 d\theta = ct, \;\; \therefore |r| = |t| \tag{0.4}$$

The use of scalar expressions for r and t is significant. If we treat r as a vector, $\boldsymbol{r}$, (we can even call the clock arm $\boldsymbol{r}$), with its origin at the center of rotation and its extension point at the circumference of the clock face, then it is clear that $d\theta$ is orthogonal to $\boldsymbol{r}$. It bears emphasizing that $\boldsymbol{r}$ is a 3-vector. The unit integral of $d\theta$ can be thought of as an operator that transforms $\boldsymbol{r}$ orthogonally into a tangent vector, $c\boldsymbol{t}$, so that $\boldsymbol{r}$ and $\boldsymbol{t}$ are seen to be essentially orthogonal to each other, stated explicitly using the orthogonal sense, i, as

$$\boldsymbol{r} = ic\boldsymbol{t} \tag{0.5}$$

with the scalar form

$$r = ct \,. \tag{0.6}$$

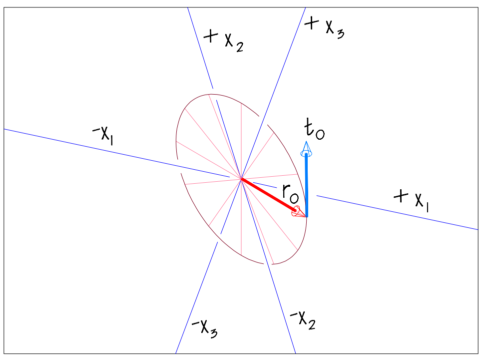
But such orthogonality, here made explicit in equation (0.5), is exactly what a dimensional relationship between space and time demands. The presence of the c is simply as a normalizing coefficient or reminder that r and t are normalized as indicated in equation (0.4), or with the use of the orthogonal sense, i, as in the normalized identity

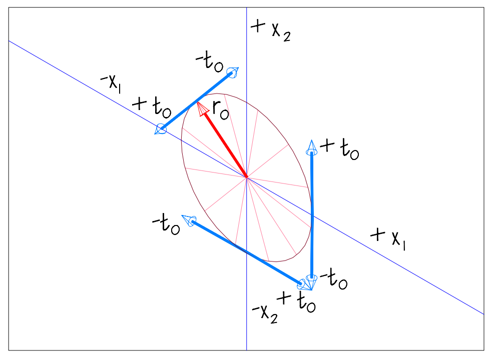
$$\boldsymbol{r} \equiv i\boldsymbol{t} \,. \tag{0.7}$$

Since $\boldsymbol{r}$ is radial and $\boldsymbol{t}$ is tangential, it is immediately apparent that in addition to being orthogonal to a spatial length, $\boldsymbol{r}$, time is periodic. It continues to cycle around the origin of $\boldsymbol{r}$, and assuming that $\boldsymbol{r}$ is set to some unit standard, $\boldsymbol{r}_0$, after a period of 2π it will return to its starting point and will continue to cycle at the invariant rate or angular frequency

$$\omega_0 = \frac{d\theta}{dt} = \frac{c}{r_0}. \tag{0.8}$$

We can envision $\boldsymbol{r}_0$ rotated to any direction in three dimensional space, with its origin translated in any direction, and the dimension of time will remain extending orthogonally, that is, tangentially from the instant point of $\boldsymbol{r}_0$, as shown in Time Scale 1.





Time Scale 1 - Clock face fixed/rotating hand **Time Scale 2 – Clock face rotates with hand**

We might think of the instant $\boldsymbol{r}_0$ as equal to a unit base vector along an instant spatial dimension x_1, for which x_2 and x_3 are the remaining instant orthogonal dimensions necessary to span a three dimensional space. Since we are limited to three spatial dimensions, $x_{i=1,2,3}$, in most graphic representations, the addition of an orthogonal dimension of time, $t = x_0$, involves representational difficulty. Note that in the last paragraph, were we to shift the origin of tangent $\boldsymbol{t}_0$ from the point to the origin of the vector $\boldsymbol{r}_0$, $\boldsymbol{t}_0$ would be co-linear with another unit vector orthogonal to $\boldsymbol{r}_0$, call it i$\boldsymbol{r}_0$, that is, to another spatial dimension, say x_2, so that in a 2 dimensional graphic representation of spacetime $t = x_2$, necessarily substituting the dimension x_0 for, or suppressing, x_2. In a 3 dimensional depiction, we might make the equation of $t = x_3$, representing physical space as a two dimensional plane, x_1-x_2. Both methods are used in discussions of general relativity, with the familiar warping of spacetime represented by a curving funnel in the 3-D depiction.

While such representation has its time tested merits, it yet depends upon the explicit relationship of equation (0.5), which in turn retains the implicit relationship of equation (0.8). We would hope, therefore, to find a representation of spacetime which can depict time as orthogonal to all three dimensions of space, without the suppression of one or two spatial dimensions. In such case, time can be though of as a compactified dimension resident on some scale, r_0, at each locus of 3-D space.

For a registration of time on a clock face, a single, vector like time hand is convenient, but some reflection will indicate that the same standard of change could be applied to the clock face as a whole. Instead of the hand moving about the face, we might imagine the hand along with the entire face rotating about some center, within the surrounding volume of space. Any spot on the circumference at a distance of r_0 from the center represents the origin of a tangent unit time vector $\boldsymbol{t}_0$, its direction either clockwise or counterclockwise. The clock face, i.e. time itself, then is moving orthogonal to two spatial dimensions, say x_1 and x_2, as shown in Time Scale 2. Note that it is moving orthogonal to $\boldsymbol{r}_0$ and orthogonal to an arbitrary x_1 and x_2 centered on the origin of $\boldsymbol{r}_0$.

Since we have stipulated above that the clock hand can be rotated or translated without changing the relationship of equation (0.5), the same can be said for a rotation or translation both in 2-D and in 3-D space of the whole clock face. Sticking to the 2-D case, in x_1-x_2, we can designate a pair of differential vectors, $\boldsymbol{dt}$, pointing clockwise and counterclockwise, at each possible location of the point of an $\boldsymbol{r}_0$ about the clock face, so that the sum of all $\boldsymbol{dt}$ forms two superimposed circles about the instant center of the clock face. The dimension of time then forms a circle of radius r_0 about each point in x_1-x_2. This can be related to a polar coordinate system, in which the arm of the clock face, $\boldsymbol{r}_0$, is a norm and the x_1-x_2 plane is sectioned as the θ coordinate about its origin.

What about a 3-D space? In x_1-x_2-x_3, we can once again designate a differential vector pair, $\boldsymbol{dt}$, at each possible location of the point of an $\boldsymbol{r}_0$ about the clock face and at each possible orientation of the clock face within the 3-space, so that $\boldsymbol{dt}$ can point anywhere in a tangential plane and so that the sum of all $\boldsymbol{dt}$ forms an infinitude of superimposed spheres about the instant center of the clock face. Thus the dimension of time, t, is orthogonal to all three spatial dimensions, x_i, of any arbitrary spatial orientation at the points x_i = +/-1.

Now we can simplify and make things a bit more definite as in Time Scale 3. For any clock face θ of radius r_0 in θ, an arbitrary x_1-x_2 plane, rotating about an axis, **θ**, aligned with the x_3 axis orthogonal to x_1-x_2, we can find a second clock face ϕ of equal r_0, concentric with, orthogonal to, and rotating with θ, i.e. spinning like a coin, while itself simultaneously rotating at the same frequency, $\omega_\phi = \omega_\theta$, about an arbitrary axis, **ϕ**, where **ϕ** rotates in θ and with θ. We can now choose a clock hand, $\boldsymbol{r}_0$, its origin at the center of the concentric clock faces, initially at x_1 at one of the two radial intersections of ϕ and θ, and rotate it with ϕ about **ϕ**, so that

1. at $t_\theta = 0$, $\boldsymbol{r}_0$ points to (0,+1,0) and $\boldsymbol{t}_{0\phi}$ points to (0,+1,+1);

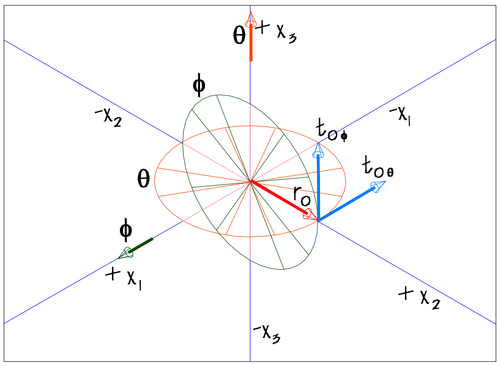
2. at $t_\theta = \pi/2$, $\boldsymbol{r}_0$ points to (0, 0, +1) and $\boldsymbol{t}_{0\phi}$ points to (+1, 0, +1);

3. at $t_\theta = \pi$, $\boldsymbol{r}_0$ points to (0,+1,0) and $\boldsymbol{t}_{0\phi}$ points to (0,+1,-1);

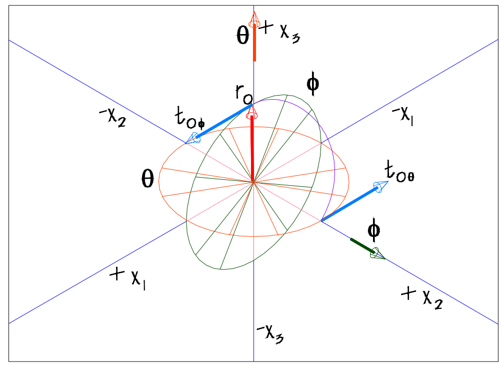
4. at $t_\theta = 3\pi/2$, $\boldsymbol{r}_0$ points to (0,0,-1) and $\boldsymbol{t}_{0\phi}$ points to (+1,0,-1); and finally

5. at $t_\theta = 2\pi$, $\boldsymbol{r}_0$ points to (0,+1,0) and $\boldsymbol{t}_{0\phi}$ points to (0,+1,+1);.

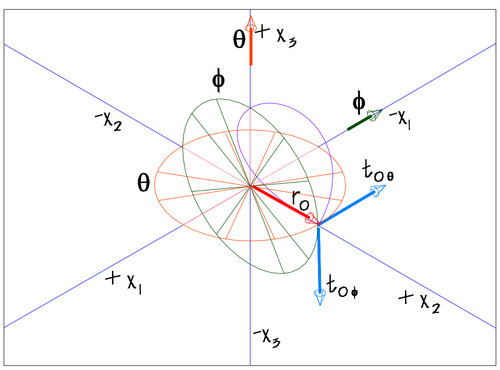
There are an infinite number of $\boldsymbol{r}_0$ in ϕ, they each intersect with the clock face of θ twice and at the same location in θ with each cycle of ϕ about $\boldsymbol{\theta}$, and they each extend once to each of the extrema in ϕ at $+/-\pi/2$. Thus the point of each $\boldsymbol{r}_0$ and the origin of its time vector $\boldsymbol{t}_0$, traces a figure eight oscillation about one half of the spherical shell formed by the sum of all time vectors $\boldsymbol{dt}$. Note that his motion avoids the coordinate entanglement condition. We can use this graphic depiction of time to great advantage later in our discussion.



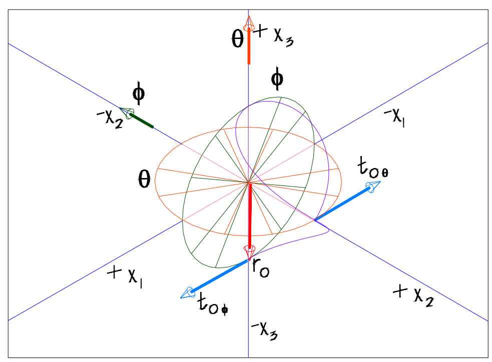
Time $t_\theta = 0$



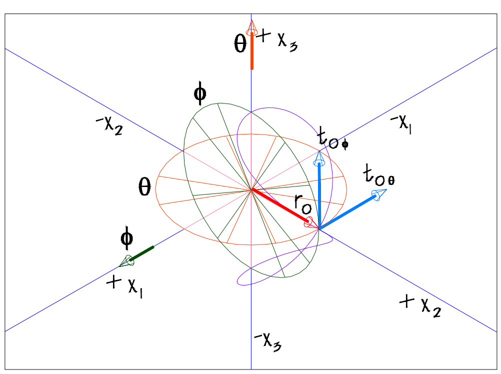
Time $t_\theta = \pi/2$



Time $t_\theta = \pi$



Time $t_\theta = 3\pi/2$



Time $t_\theta = 2\pi$

Time Scale 3 – Clock face rotates with hand and spins on edge at common frequency

Note that the same instance of time is represented anywhere on the spherical surface of this clock, so that the surface constitutes a co-ordinate singularity. We can keep track of the "length" of time by a count of the number of oscillations of a given $\boldsymbol{r}_0$. We might also envision that the length of $\boldsymbol{r}_0$ is in some manner augmented or diminished by a very small amount continually with each oscillation, so that the time dimension is seen to be wound up in the manner of a kite string about a constantly increasing or decreasing spatial unit sphere. It is important to remember, however that there are an infinite number of such $\boldsymbol{dt}$ continually connected in spherical fashion, so the string analogy should not be stretched too far. It is really the expanse of 3-space both about and within such unit sphere, expanding or contracting, that marks the passage of time. It is the expansion of this space at the speed of light, but tangentially and not radially, that gauges time in this spacetime.

Lorentz Covariance

To complete this analysis, we would like to see if this formulation is Lorentz covariant, if the standard of time, t_0, so designated will undergo a scale transformation along with the length standard, r_0, according to the principles of special relativity. Returning to equation (0.5), we might envision that under some condition, such as the acceleration due to cosmic expansion, r_0 contracts to $r_0{}^o < r_0$. We divide that equation into its contracted version,

$$\frac{r_0{}^o}{r_0} = \frac{ict_0{}^o}{ict_0} = \frac{t_0{}^o}{t_0} \tag{0.9}$$

and find the unit time standard varies according to the ratio of the unit lengths, as

$$t_0{}^o = \frac{r_0{}^o}{r_0} t_0 . \tag{0.10}$$

In special relativity[2], time intervals transform according to

$$t' = \gamma\left(1-\beta\right)t \tag{0.11}$$

where t is the interval in reference frame F and t' is the same interval viewed in reference frame M moving relative to F at velocity, v, as a fraction of the speed of light, c, giving the ratio identity β, which cannot be greater than 1, as

$$\beta \equiv \frac{v}{c} \tag{0.12}$$

and the value of γ, which cannot be less than 1, as

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} . \tag{0.13}$$

Equation (0.12) approaches 0 faster than the inverse of equation (0.13), so the combined factor never exceeds 1 and approaches 0 at the limit. If a relationship can be established between the time dimensions in equations (0.10) and (0.11), then we might expect a relationship between the factors on the right sides. We can do this by viewing a unit standard, t_0, from F and from M.

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[2] As developed in Stevens, Charles F., 1995, The Six Core Theories of Modern Physics, (Cambridge: The MIT Press) p. 183, among many other references.

The spatial interval transformation, in which we have aligned r with an arbitrary x_i axis, is

$$r' = \gamma(r - vt) \tag{0.14}$$

Substituting from equation (0.6) for t, we have

$$r' = \gamma\left(r - v\frac{r}{c}\right) = \gamma(1-\beta)r \tag{0.15}$$

which is symmetric with equation (0.11).

Rearranging gives an expression of a proper time, τ, and a proper length[3], σ, which are invariants of M,

$$\tau \equiv \frac{t'}{\gamma} = (1-\beta)t \tag{0.16}$$

$$\sigma \equiv \frac{r'}{\gamma} = (1-\beta)r \tag{0.17}$$

In the Chart 1 graphic representation of a Lorentz transformation we have aligned the spatial axis, r, of a stationary reference frame, F, with the direction of travel of a moving frame, M, making it a pure transformation or boost. This is expressed for the time dimension by equation (0.11) and for the space dimension by equation (0.15). In each equation, the unprimed coordinate with respect to F is modified by the two related factors, (1-β) and γ, to arrive at the primed co-ordinate with respect to M.

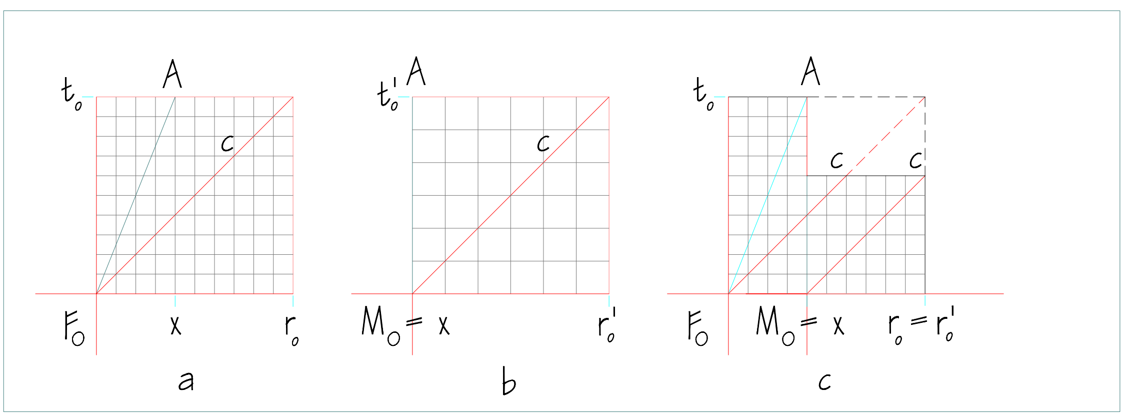

Chart 1 - (1-β) Component of Boost

While the customary analysis is for an arbitrary x, or in this case, r and t in F, we will apply the same to a space and time unit length in F, r_0 and t_0, orthogonally aligned. This is shown in Chart 1.a. The path of M, moving at a velocity v, or a distance of x in time t_0, is drawn as the sloped line, and terminates at point $A = M(x,\ t_0)$. The unit of

[3] We are not using Minkowski space for our 4-vector and r is simply ct, so that multiplying equation (0.16) through by c gives us equation (0.17). This proper length will be shown to be related to $r_0^{\ o}$.

spacetime has been marked off in decimal fractions. The normalized speed of light as a limit of relative frame velocity is shown with its inverted slope of 1 t_0 per 1 r_0.

In Chart 1.b, the operation of (1-*β*) on *F* indicates the effect of the motion of *M*, which transforms the unit spacetime from that shown in 1.a. Assume that both *F* and *M* begin to receive a periodic signal from beyond the left edge of their respective charts when those charts are coincident at $r = r' = 0$. They both know that the signal flashes are spaced one-tenth of t_0 apart. As shown, *x* and therefore *β* happens to be 0.4, resulting in a (1-*β*) of 0.6. After one t_0, *F* counts ten flash intervals, but *M* has by that time moved four intervals to the right and only counts six intervals. As a result, for *F* the perceived time elapsed before the first signal reaches r_0, therefore the distance from 0 to r_0 is ten-tenths or unity, while for *M* that time and distance is six-tenths of t_0 and r_0 respectively.

Note that the path of *M* observed from *F* in Charts 1.a and 1.c, the diagonal through space and time, is perceived by *M* in his own view of this spacetime, as simply a path through time, shown by the vertical blue line. Note also that the shortening of the time scale is required if *c* is to remain normalized and invariant.

This is not the time dilation and space contraction of relativity, however. If the signal had been coming from the right, during the time t_0, *M* would have counted fourteen intervals to a count of ten for *F*, or a factor of (1+*β*). This is simply an instance of the Doppler effect, a frequency shift.

As can be seen in Chart 1.c, the gauge or scale factor of the spacetime is the same in both frames, as indicated by the identical grid intervals. The unit time and distance scales of the spacetime for each are not themselves modified by this observed modification, and we will disregard it in the remainder of the discussion. It is of interest, though, that the product of these two factors equals the square of the inverse of the other factor, *γ*, or

$$\left(1-\beta\right)\left(1+\beta\right) = \left(1-\beta^2\right) = \sqrt{1-\beta^2}^{\,2} = \gamma^{-2}. \qquad (0.18)$$

It is the factor γ that we are primarily interested in, as it embodies the change in the scale of spacetime reflected in a measured interval through the Fitzgerald-Lorentz length contraction,

$$\gamma\ \Delta r = \Delta r' \qquad (0.19)$$

and through time dilation,

$$\gamma\ \Delta t = \Delta t'. \qquad (0.20)$$

These in turn would appear to be related to a change in the proper time, *τ*, and proper length, *σ*, as in equations (0.16) and (0.17) as

$$\gamma d\tau = \gamma dt = dt' \qquad (0.21)$$

$$\gamma d\sigma = \gamma dr = dr' \qquad (0.22)$$

Following this line of thought, we substitute the unit standards for the un-indexed interval coordinates in equations (0.16) and (0.17) to arrive at an expression of a unit proper time,

τ_0, and a unit proper length, σ_0, where each is the representation of the unit standards of *M* in *F*,

$$\gamma\ \tau_0 \equiv t_0^{\,o} = \gamma\left(1-\beta\right)t_0 \tag{0.23}$$

$$\gamma\ \sigma_0 \equiv r_0^{\,o} = \gamma\left(1-\beta\right)r_0 \tag{0.24}$$

Some care is in order here. While the length contraction is often interpreted as a property by which a moving body shrinks absolutely in proportion to its velocity with respect to a stationary frame, and while this may in some instances be true, its fundamental statement is that the unit standard by which a length, *l*, is measured in a moving frame is smaller than the unit standard in the stationary frame with respect to which it is deemed to be moving and from which it is held to be shorter.

In a similar manner, time dilation is deemed to indicate that a given duration of time in a moving frame is measured as moving slower from a stationary frame; thus the usual depiction of the space traveler who returns to earth after 50 years of near speed of light travel, having aged only a couple of earth years. As in the last paragraph, equation (0.20) states the same physical condition as equation (0.19), that the unit standard of time in a moving frame is smaller than the unit standard in a stationary frame, thus a duration of time is measured as greater, i.e. longer as is a length, in the moving frame, but this does not necessarily mean slower.

If our clocks in both the moving and the stationary frame are defined as having hands of a length measured by equation (0.19), and the speed of the end of the hand is the speed of light, *c*, then the moving frame will have a longer arm and its angular velocity will necessarily be less than that of the stationary frame, and the clock in *M* will rotate at a slower rate than in *F*. This is the general interpretation of time dilation. On the other hand, if the length of the hand in *M* is set to the unit length standard, smaller in *M* than it is in *F*, then the speed of light constraint for the speed of the hand tip will result in an increased angular speed and the clock in *M* will spin faster. In such case time will still be measured as greater, i.e. longer in *M* than in *F*, as a count of the number of clock cycles would indicate, in keeping with equation (0.20), since γ in this case is a measure of the relative angular frequencies of *M* and *F*. This is so even though the length of the clock hand path in keeping with equation (0.8) is the same, or

$$r_0\omega_0 = r_0^{\,o}\omega_0^{\,o} = c \tag{0.25}$$

since

$$\frac{r_0}{r_0^{\,o}} = \frac{\omega_0^{\,o}}{\omega_0} = \frac{c}{r_0^{\,o}\omega_0} = \gamma\ . \tag{0.26}$$

With this in mind, we can combine equations (0.20) and (0.19) as we did in equation (0.9), converting from incremental to differential values, and get the equivalent of equation (0.10), where this last case explicitly shows the equivalence of the differential length ratio and γ,

$$dt' = \frac{dr'}{dr}dt = \gamma dt\ . \tag{0.27}$$

We have a temporary conundrum, however, as $\gamma \geq 1$, but the unit length ratios in equation (0.10) and again if inverted from equation (0.26) is less than 1. The problem arises from the nature of a unit standard. If it is fixed, any change in an interval, differential or incremental, will vary directly, proportional to the standard. If the standard itself varies, then the numerical value of a fixed interval will vary indirectly to the change in the standard.

Given a fixed interval, $l \equiv l'$, which is related nominally by γ as measured from frames *M* over *F*, equation (0.27) measures the identical interval, $dt \equiv dt'$ from two different physical standards. Equation (0.10) relates two unit standards, $t_0 > t_0^{\,o}$, that vary proportionally to the two other unit standards, $r_0 > r_0^{\,o}$, all related by *c*. Thus

$$\frac{l}{r_0}\gamma = \frac{l'}{r_0^{\,o}} \tag{0.28}$$

$$\therefore \gamma = \frac{r_0}{r_0^{\,o}} \tag{0.29}$$

We return now to the charts to see how this might be represented graphically. Chart 2 shows an enlargement of the top portion of Chart 1.a in the neighborhood of the time t_0 in *F*. We are analyzing only the effects of the factor γ on the two reference frames and disregarding the Doppler effect of (1-β). Point *A* represents the intersection of the line of motion of *M* in *F* and the time coordinate in *F* for time t_0, designated as $F\left(x, t_0\right)$. In reference frame *M*, based on the above discussion and equations (0.19) and (0.20), this same point would be measured as $M\left(\gamma x, \gamma t_0\right)$, which as drawn for $\beta = 0.4$, so that $\gamma = 1.0910\ldots$, would be (0.4364…, 1.0910…). Finally, based on these same two equations this point is expressed as the intersection of x' and t_0', as shown in the square brackets or $M\left(x', t_0'\right)$.

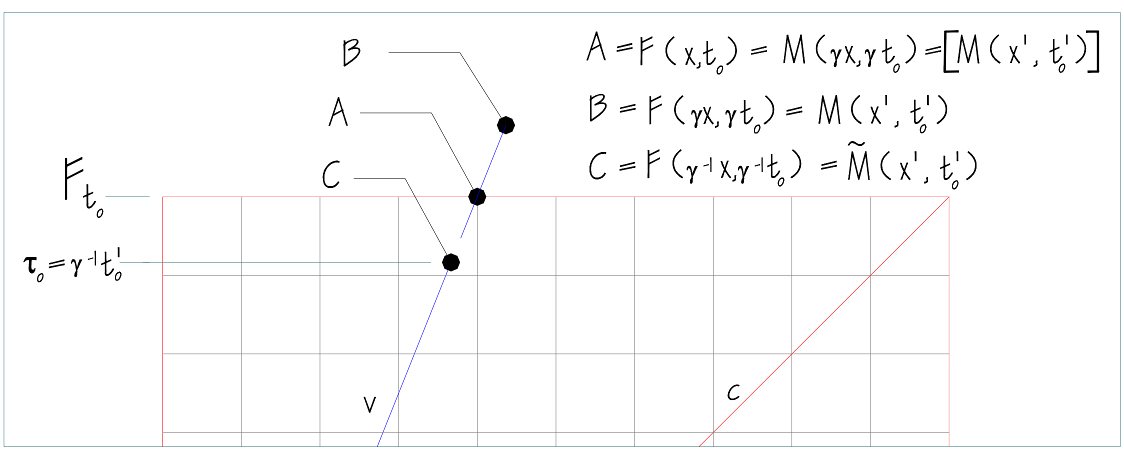

Chart 2 – γ Component of Lorentz Transformation

Point B shows the co-ordinates in F corresponding to the numerical values of $M(\gamma x, \gamma t_0)$, and therefore represents an expansion of the line for v by the value of γ. Thus it expresses M in terms of F and is numerically equal to the value $M(x', t_0')$.

Point C shows the numerical value in terms of F for the inverse of $M(x', t_0')$ or $\tilde{M}(x', t_0')$. Thus if we were to designate $M(x', t_0')$ in M as $M(0.4, 1.0)$, $\tilde{M}(x', t_0')$ would be $F(0.366..., 0.916...)$. The time component of C then represents the *proper time*, τ_0, the naught subscript used to indicate its specificity to a unit time standard, t_0', of M, when measured from F, and in keeping with the concept of time dilation, it is longer in M than in F. Thus for a value in M of $t_0' = 1$, F will perceive an elapsed time in M of $\tau_0 = 0.916...$. Once again, while generally interpreted as a slowing of time in M, this "lengthening" of time can be attributed to a shortening of the time standard.

This is all very interesting, but it would be more illustrative if we could find an essential depiction of the relationship of F and M involving γ and τ_0. For instance, the length of v from F_0 to the three points, A, B and C, embodies the factor of γ, but that factor does not arise naturally, or at least readily, from an analysis of the charting of v.

The problem lies in the dual utility of the chart itself. On the one hand it represents a Cartesian background for the plotting of two related bits of data, location in time and in space. From this perspective, the right hand end of the speed of light curve, c, at the upper right corner of the chart, represents the time elapsed in F during the displacement of a light wave or photon by one unit, or $F(r_0, t_0)$. On the other hand, it is a 2-D chart of spacetime itself, where the speed of light determines the unit speed for the passage of a stationary reference frame through time or of a displacement through space with no passage of time. This second usage means that in time t_0, the limit of travel of a spacetime vector in the unit spacetime is a circle, or in our chart, a quarter circle, described by the unit spacetime vector, R_0, as distinguished from the space vector, r_0, or

$$R_0 = \left(r^2 + t^2\right)^{\frac{1}{2}}. \tag{0.30}$$

We will use the designation R_0 for both the vector and the circle described using it as a radius, dependent on context. When R_0 is orthogonal to the time axis it is a pure space vector and equals r_0 and when orthogonal to the space axis is a pure time vector and equals t_0. It should be mentioned that in a 4-D spacetime, R_0 is an invariant 4-vector, but that it is not the same 4-vector residing in Minkowski space, as generally used in relativity, as the time vector is not subtracted, but rather is added to the three space vectors, as in equation (0.50). While the sum of the four vectors in Minkowski space results in a null 4-vector, in this case the addition results in a unit 4-vector.

Drawing this condition on the unit spacetime for *F* gives us Chart 3.a, and we notice immediately that the velocity curve used for the moving frame *M* terminates at *A*, beyond the limit imposed by *c*; that is, it violates one of the basic assumptions of relativity. To correct this, in Chart 3.b we draw the velocity curve, v^o, through the intersection of R_0 and x_1 at A^0, as shown in close-up in Chart 4, and find on closer inspection that this corresponds with the time value of τ_0 for x_1. In fact, for any value of $0 < x < r_0$, this condition will be found to hold, which means that the secant of the angle between t_0 and v^o equals γ, or

$$\frac{\overline{OB^o}}{t_0} = \frac{\overline{OA^o}}{\tau_0} = \gamma \, . \tag{0.31}$$

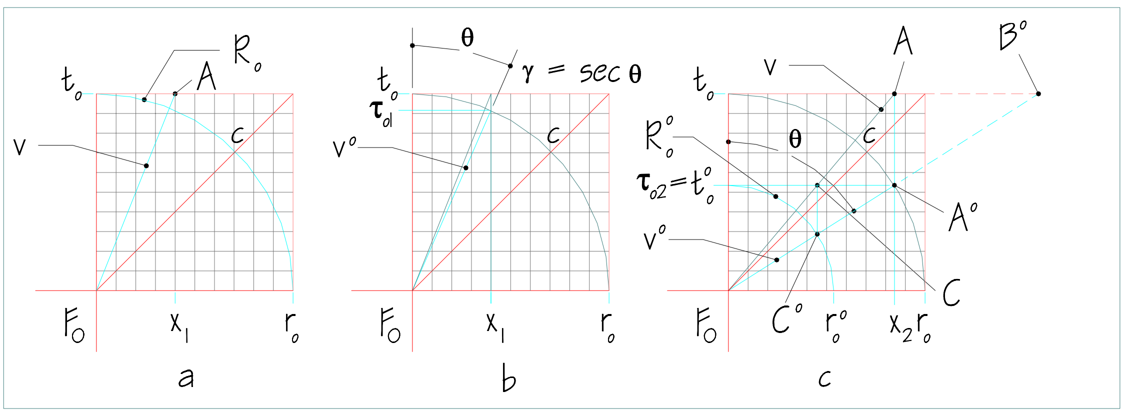

Chart 3 – Contraction of γ Component of Lorentz Transformation

Chart 3.c, a condition at a much higher velocity, shows more clearly the relationship of v^o and τ_0. We construct a second circle for $R_0{}^o$ such that $\left|R_0{}^o\right| = \left|r_0{}^o\right| = \left|t_0{}^o\right|$, where

$$t_0{}^o \equiv \tau_0 \tag{0.32}$$

The orthogonal projection of the intersection of R_0 and v^o onto τ_0 intersects the curve *v* at point *C*, while an orthogonal projection from *C* onto $R_0{}^o$ intersects $R_0{}^o$ and v^o at the same point, C^o. Thus we have the similar triangles, $B^o A A^o \sim A^o C C^o$, and $R_0{}^o$ represents a contraction in the moving frame of the unit spacetime vector R_0.

Chart 4 is a close-up view of the top portion of Chart 3.b, showing the contraction of *v* into v^o. We will call this reference frame F^o. A^o represents the same physical condition as *A* in *F*. The displacement *x* remains as the same percentage of r_0, but the time scale at that point is now the proper time, τ_0. In square brackets, that same point in M^o is numerically identical to $M(\gamma x, \gamma t_0)$ at *A*, however, the unit time standard, $t_0{}^o = 1$ and the related x^o, are in M^o instead of *F*. Thus the unit standard is local to M^o instead of the expression of a stationary spatial background. If we can think of the time dimension M_t^o as being inclined along the slope of v^o instead of orthogonal as in F, the same spacetime numerical values hold in both reference frames.

In keeping with this observation, C^o is proportionally the same to A^o as C is to A. In M^o, the unit time and space standards apply as indicated by $R_0{}^o$, so that $M^o\left(x^o, t_0{}^o\right)$ is numerically equal to $F\left(x, t_0\right)$ shown at A, or in this case, (0.4, 1.0). For the proper time, with all values in units of F, we have the following identity,

$$\tau_0 \equiv \gamma t_0{}^o \equiv \gamma^{-1} t_0 \tag{0.33}$$

Point B^o indicates the nature of time dilation as conventionally figured. At point A^o, M^o has traveled the same length of time as F, as given by $R_0 = t_0$, but to F this is registered as the proper time τ_0. By the time M^o reaches B^o, which F registers as 1.0, F has moved on in time to γt_0. The length of v^o, $\overline{OB^o}$, is equal to γ.

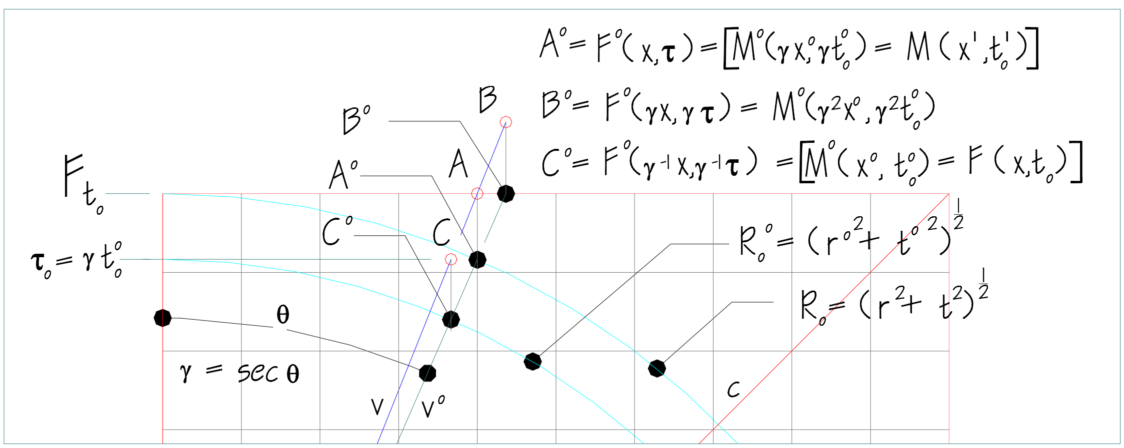

Chart 4 – Close-up of Contraction

Perhaps the most significant aspect of this representation is that the secant of the angle of $t_0 \, \mathrm{O} \, v^o$ establishes γ, underscoring the geometric nature of time. Expanding on the relationships of equation (0.31), we have

$$\frac{\overline{OB^o}}{t_0} = \frac{\overline{OA^o}}{\tau_0} = \frac{R_0}{R_0{}^o} = \frac{t_0}{\tau_0} = \frac{\tau_0}{t_0{}^o} = \frac{\overline{OB^o}}{\overline{OA^o}} = \frac{\overline{OB}}{\overline{OA}} = \frac{\overline{OA}}{\overline{OC}} = \gamma \tag{0.34}$$

We have examined the Lorentz transformation with respect to time and proper time, but a similar analysis could be made with respect to space and a proper distance as modified from the conventional Minkowski representation as noted earlier. The case of time is more germane to our present discussion, as will be seen.

The above charts suggest that spacetime curvature is as much a matter of curvature of time as it is of space. Chart 3 indicates that as a moving reference frame approaches the speed of light and v approaches co-linearity with c, v^o and γ approach infinity and co-linearity with the space axis, r, and the time and distance scales indicated by $R_0^{\ o}$ become exceedingly small, and in the same proportion. This is precisely what we analyzed initially with equation (0.5) and a cyclical time dimension. If we envision M as an accelerating reference frame starting from rest at F_O and accelerating to c within the first unit of spacetime, we would find that the contracted velocity curve, v^o, and the collinear contracted time dimension would curve, and that under certain constraints, would arc like a quarter circle. Its constantly shortening time standard, $t_0^{\ o}$, then is aligned with the arc of γ, and its space standard, $r_0^{\ o}$, correspondingly shortened, rotates with and orthogonally to it.

A physical instance of this shortening of both $r_0^{\ o}$ and $t_0^{\ o}$ can be found by examining the nature of the deBroglie wavelength of a massive particle. We assume that the reduced Compton wavelength, $ƛ_C$ is indicative of the rest state of such a particle, and is determined by dividing the reduced Planck's constant, $\hbar$, by the product of the speed of light and the particles rest mass, m,

$$ƛ_C = \frac{\hbar}{mc}. \tag{0.35}$$

The reduced deBroglie wavelength, $ƛ_{dB}$, is the quotient of $\hbar$ and the particle's relativistic momentum, p, at velocity, v, given as

$$ƛ_{dB} = \frac{\hbar}{p} = \frac{\hbar}{\gamma m v} \tag{0.36}$$

where the factor γ is the same as used in the development above. Combining and some rearrangement gives us the ratio of these reduced wavelengths as

$$\frac{ƛ_C}{ƛ_{dB}} = \gamma\frac{v}{c} = \gamma\beta = \gamma x. \tag{0.37}$$

In Chart 4, this is represented by the tangent of angle θ between the time axis in F and v^o at point A^o and gives the ratio of the particle velocity in F, where $A^o(x) = A(x)$, and the contracted unit standard, $r_0^{\ o}$. Thus

$$\frac{ƛ_C}{ƛ_{dB}} = \frac{x}{r_0^{\ o}} = \frac{x}{t_0^{\ o}} = \frac{x}{R_0^{\ o}} \tag{0.38}$$

and as we approach the limit of the speed of light and x approaches $r_0 = 1$, we have

$$ƛ_C = \frac{r_0}{r_0^{\ o}} ƛ_{dB} = \gamma\, ƛ_{dB} \tag{0.39}$$

Thus, in the extreme case

$$\text{if } ƛ_C = r_0\text{, then } ƛ_{dB} = r_0^{\ o}. \tag{0.40}$$

Since the frequency and wavelength are related as

$$ƛ_C \omega_C = ƛ_{dB}\omega_{dB} = c \tag{0.41}$$

rearrangement gives, in the extreme case

$$\gamma = \frac{\lambda_C}{\lambda_{dB}} = \frac{\omega_{dB}}{\omega_C} = \frac{t_C}{t_{dB}} = \frac{r_0}{r_0{}^o} = \frac{\omega_0{}^o}{\omega_0} = \frac{t_0}{t_0{}^o}, \tag{0.42}$$

therefore we also have

$$\text{if } t_C = t_0\text{, then } t_{dB} = t_0{}^o. \tag{0.43}$$

Considering a normalized frequency, that is, where the angular displacement, $\theta = \theta_0$, always equals 1 and the time consequent varies according to the particular *F* from which it is observed, we can integrate equation (0.25) for any time $t = qt_0 = q^o t_0{}^o$

$$r_0\omega_0 \int_0^t dt = r_0{}^o\omega_0{}^o \int_0^t dt \tag{0.44}$$

$$r_0\omega_0 \left(qt_0\right) = r_0{}^o\omega_0{}^o \left(qt_0\right) = r_0{}^o\omega_0{}^o \left(q^o t_0{}^o\right) \tag{0.45}$$

$$qr_0 = q\gamma r_0{}^o = q^o r_0{}^o \tag{0.46}$$

and finally

$$r_0 = \gamma r_0{}^o = \frac{q^o}{q} r_0{}^o \tag{0.47}$$

therefore

$$t_0 = \gamma t_0{}^o = \frac{q^o}{q} t_0{}^o \tag{0.48}$$

showing that γ is simply the frequency ratio of the unit standards of space and time between a moving and a stationary reference frame.

It is worth noting, that this ratio is unity when $x = R_0^o = r_0{}^o = \frac{1}{\sqrt{2}}$, that is, when *r* equals *t* at the intersection of the curve of *c* and R_0. If we consider a massive particle as some manner of physical stationary waveform, i.e. a bound, rotating wave, a ratio of unity represents the point at which the translational displacement of the particle in space begins to outrun the transverse wave displacement, i.e. its displacement in time. It is the point at which the contracted velocity, v^o, equals the speed of light. Prior to that point the waveform could conceivably flatten in space in the form of an oblate spheroid. From that point on, the waveform becomes prolate or contracts in all dimensions to keep from outrunning itself.

It follows immediately that from any reference frame *F* in 4-D spacetime, for a moving frame *M*, a unit standard can be given for space by $r_0{}^o$ and for time by $t_0{}^o$, both related to a 4-vector (of additive components), $R_0{}^o$, as

$$R_0{}^o = \tfrac{1}{\sqrt{2}}\left(\left(r_0^o\right)^2 + \left(t_0^o\right)^2\right)^{\frac{1}{2}} = \frac{R_0}{\gamma} \tag{0.49}$$

where

$$R_0 = \tfrac{1}{\sqrt{2}}\left(r_0{}^2 + t_0{}^2\right)^{\frac{1}{2}} = \tfrac{1}{\sqrt{2}}\left(\left(\tfrac{1}{\sqrt{3}}x_{01}\right)^2 + \left(\tfrac{1}{\sqrt{3}}x_{02}\right)^2 + \left(\tfrac{1}{\sqrt{3}}x_{03}\right)^2 + t_0{}^2\right)^{\frac{1}{2}}. \tag{0.50}$$

If we shift the origin of $\boldsymbol{t}_0^o$ in Chart 3 from the origin of $\boldsymbol{r}_0^o$ to its point, we have the configuration shown in Time Scale 1 and 2. From there we can extrapolate to the 3-D form shown in Time Scale 3 for the expression of a 3 dimensional clock.

A statement is in order concerning the "relativity" of the reference frames *F* and *M*, and that of the spacetime scales R_0 and $R_0{}^o$. Assuming that *F* resides in an expanding 3-manifold, if that residence is isotropic with respect to cosmological red shift, then we can state that the local position of *F* is stationary with respect to space and in an extremal position of change with respect to time. Otherwise, *F* would experience a blue shift in the direction of its travel with respect to space. In similar fashion, *F* could experience such an anisotropy while rotating about a center, perhaps galactic or supergalactic, that is itself stationary or isotropic with respect to cosmological red shift. Thus at every point in spacetime, assuming an isotropic expansion, there exists a potential *F* for which R_0 is a local maximum, though R_0 at all points need not be identical. For any moving reference frame *M* at that same point, $R_0^o < R_0$.

Thus we can envision a 4-D spacetime with Lorentz covariance in which the time dimension is modeled as a compactified rotational dimension orthogonal to the three space dimensions, as developed earlier. Having taken this side-trip into an investigation of spatial and temporal length, we can now look at the concept of mass.

1 – Geometrization of Mass in Classical and Quantum Theories

In his book, Concepts of Mass in Contemporary Physics and Philosophy, Max Jammer delineates three types of mass[4]; inertial, active gravitational (corresponding to a source) and passive gravitational (corresponding to a test particle), and concludes that the jury is still out as to whether these represent distinct concepts of mass. Looking at the related concept of inertia, we can readily see that it can be quantified in terms of length and time by the concepts of displacement and velocity. For simplicity, we limit our thought experiments to analysis in one spatial dimension, unless stated otherwise.

Inertial Mass

Inertia is "a resistance to any change in the momentum of a body"[5].

1. An absolute or infinite inertia would indicate immobility or a displacement of $dx = 0$ from the reference frame of that body or a change in velocity of $dv = 0$ from any arbitrary reference frame, resulting from interaction with another body.
2. An absolute lack of or zero inertia would indicate an instantaneous displacement of *a*, an undefined or relative infinite displacement or change in velocity due to a finite displacement with zero passage of time.
3. A finite displacement of a body, *a*, over a finite time duration resulting from its interaction with another body is a measure of its finite inertia, i.e. of its inertial mass.

While "body" has been historically conceived as a classical entity, substitution of the term "particle" understood in quantum terms, should not change the meaning of "inertia". A free body or particle is generally conceptualized as moving within and through a flat, three dimensional space, which of itself and in the absence of any field potential or other bodies or particles of either mass or energy, constitutes both a phenomenological and an ontological void. By the above definition and our expansion of it, however, a space upon which we can superimpose a metric, in and of itself exhibits the property of inertia, since it has a definite resistance to change and in the case of physical space, appears to have a finite, albeit accelerating, expansive momentum as evidenced by cosmological red-shift. By virtue of such property, space even without quantum fields can not be said to be either a phenomenological or an ontological void. Within such space, time can be seen as the path of its inertial change.

In the interest of gaining a geometric, descriptive explanation of mass, we will investigate inertial mass first as a classical target or test body or particle. For body *a* the magnitude of its mass, m_a, is indirectly proportional to the displacement, x_a, over the time interval of an interaction, t_a, under a given impulse, *J*, from another body or source, which results in

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[4] For the etymologically inclined, *mass* is from the German *massieren* meaning *to knead dough*, and evokes the notion of folding and stretching the dough with the heel of the hand, turning it 90^o, and repeating the process, which forms gluten, allowing the dough to catch the gas of the leavening agent and expand.

[5] Lapedes, Daniel N., Editor in Chief 1978, McGraw-Hill Dictionary of Physics and Mathematics, (New York: McGraw-Hill Inc.)

a final velocity for *a* of v_a, and is therefore directly related to the time interval per that displacement,

$$m_a = \frac{J}{v_a} = \tfrac{1}{2} J \frac{t_a}{x_a}. \tag{1.1}$$

In general relativity, mass is geometrized in direct relationship to length, and we can find a direct relationship between mass and length in the aggregation of bodies or particles. As in the case of stellar configurations, the product of the volume of the body and its average volume inertial density computes the mass of the body, so that for a given density, i.e. the number of quanta times the average mass per quanta per volume, the reduced circumference of the body gives a geometrized approximation of its mass.

The definition of impulse[6] is the integral of force with respect to time which is equal to a change in momentum, ΔP,

$$\boldsymbol{J} = \int_{t_i}^{t_f} \boldsymbol{F}(t)\, dt = \Delta P. \tag{1.2}$$

While in general the force, hence the acceleration, will vary over the duration of the impulse, for ease of illustration, we will use a constant force and acceleration, i.e., the average over the duration. In this case *a* is accelerated from an initial velocity, v_{ai}, to a final velocity, v_{af}, over the time interval $t_\Delta = t_f - t_i = t_a - t_0 = t_a$. The time subscripts indicate that at time $t_i = t_0$, $v_{ai} = 0$. Starting at the end of such interaction, at time $t_f = t_a$, the final velocity of *a* will be reached at $v_{af} = v_a$, and it will continue on at that velocity in its original reference frame, *F*.

We assume that the source of the impulse and the test body exist in a steady state in their respective rest frames and in isolation from each other and any other interactions both before and after their collision, but that during their interaction they each undergo an acceleration and an exchange of momentum and energy. Thus the acceleration for *a* is

$$a_a = \frac{v_{af} - v_{ai}}{t_a} = \frac{2x_a}{t_a^2} \tag{1.3}$$

and the force is

$$F = m_a \frac{2x_a}{t_a^2} \tag{1.4}$$

Since the time interval for the acceleration of a body and the time interval found in the statement of its velocity is the same as the interaction interval, t_f, we have the following time independent parameter of an interaction

$$\text{ח} = \int_{t_i}^{t_f} \boldsymbol{J}(t)\, dt = \int_{t_i}^{t_f} \boldsymbol{F}(t)\, t_f \; dt = \tfrac{1}{2} \boldsymbol{F}\, t_f^2 = m\, \boldsymbol{x}_f \tag{1.5}$$

6 Halliday & Resnick, 1988, Fundamentals of Physics, (Ext 3rd ed.; New York: John Wiley & Sons, Inc.)

where the letter ת (tav) is an inertial constant of the interaction, of mass-length dimensions. Equation (1.1) can then be expressed in a time independent scalar form where mass is the inverse measure of the space interval of the interaction,

$$m_a = \frac{ת}{x_a}. \tag{1.6}$$

We can postulate a second condition, with J and ת unchanged, for a body b, for which

$$m_b > m_a. \tag{1.7}$$

Therefore, we have

$$m_b = \frac{J}{v_b} = \tfrac{1}{2} J \frac{t_b}{x_b} = \frac{ת}{x_b} \tag{1.8}$$

and the following inequality

$$v_b < v_a. \tag{1.9}$$

This indicates that

$$x_b \leq x_a \tag{1.10}$$

and would suggest that

$$t_b \geq t_a \tag{1.11}$$

however, the time intervals cannot be equal if $x_b = x_a$. That is, the lower velocity of v_b may be the result of a smaller displacement, a longer interaction time, or both. In any case, the inverse velocity will be greater for v_b, so that for a test body or particle, mass is an inverse measure of the displacement and a direct measure of the inverse velocity of the interaction, and a geometrized mass should reflect that kinematic relationship.

It is of interest that if we consider a source for our impulse above from a classical body, A, of mass, M_A, where

$$M_A \gg m_a, \tag{1.12}$$

moving with an initial velocity of V_A, prior to the interaction with a, we find the interaction conforms to the following equation

$$v_a = \frac{2M_A}{\left(M_A + m_a\right)} V_A. \tag{1.13}$$

Therefore, at the extreme,

$$v_a \approx 2V_A \tag{1.14}$$

and the final velocity of the test body is principally a function of the source velocity and not of the source mass. We would expect that a similar relationship would hold, if instead of representing a source in an elastic collision, M_A represented a gravitational source. If gravitational and inertial mass are equivalent, then $M_A\ V_A$ represents the impulse generated by that source, and the final velocity of a test particle a is limited by equation (1.14). Thus if V_A is limited by the speed of light, c, then v_a will be limited to $2c$. While this appears to be a violation of the postulates of relativity, it is precisely what is predicted by general relativity at the horizon of an extreme Kerr black hole.

Returning to our test body, now assuming it to be quantum, we see that equation (1.5) is related to the action, S, of the interaction, using Maupertuis' principle, by

$$S = \int_{x_i}^{x_f} \boldsymbol{J}(x) \cdot d\,\boldsymbol{x} = \frac{2m}{t_f} \int_{x_i}^{x_f} \boldsymbol{x}_f \cdot d\,\boldsymbol{x} = \frac{2m\,\boldsymbol{x}_f}{t_f} \cdot \frac{\boldsymbol{x}_f}{2}$$
$$= m\,\boldsymbol{x}_f \cdot \frac{2\,\boldsymbol{x}_f}{2t_f} = \text{ת} \cdot \frac{\boldsymbol{v}_f}{2} = \text{ת} \cdot \boldsymbol{c} = \text{ת}\omega \cdot \boldsymbol{x}_f = \hbar \quad . \tag{1.15}$$

As S is likewise an invariant of the interaction, and m and x are inversely related, so t must be inversely related to m as well (and directly related to x). Inverse time is the expression of a rate or in this case unit frequency of interaction, so that mass is the dynamic representation of the kinematics of that unit or angular frequency of the interaction, which varies in proportion to the mass and

$$\omega_b > \omega_a \,. \tag{1.16}$$

Equation (1.15) indicates that the ratio of mass to frequency is proportional to the ratio of the inertial constant and the final velocity of the body. If S and ת are both invariants, then so must be v_f, and with some substitution and rearrangement we have

$$m = \frac{\text{ת}}{\frac{1}{2} v_f} \omega = \frac{\text{ת}}{c} \omega \,, \tag{1.17}$$

where

$$c = \frac{v_f}{2} = \frac{2x_f}{2t_f} = \frac{x_f}{t_f} \tag{1.18}$$

This observation concerning the invariance of the final speed in equation (1.17) is initially something of a mystery, since it appears to violate the initial premise of equation (1.1), though it echoes the comments above with respect to equation (1.14) in the context of an extreme Kerr black hole. The final velocity v_f as derived above is a linear velocity resulting from the acceleration of a body from zero in an elastic collision, and the final displacement rate is therefore twice the actual displacement, x_f, that occurs during the interaction. In the quantum case, assuming an inelastic collision in which the kinetic energy of the impulse is transformed into the spin energy of the target, the angular frequency increases, but the radius of gyration decreases and the value of c remains constant.

From this we have the following expressions for the impulse,

$$J = \Delta P = 2mc = 2\text{ת}\omega \tag{1.19}$$

which clearly states mass as frequency and suggests a quantum interpretation, since by multiplying through by $\frac{1}{2}c$, (differentiating with respect to time and integrating with respect to displacement), we have the mass-energy relationship

$$E = \tfrac{1}{2} Jc = mc^2 = \text{ת}c\omega = \hbar\omega = \frac{\hbar c}{\lambda} \,. \tag{1.20}$$

It follows that

$$\text{ת} = \frac{\hbar}{c} \,. \tag{1.21}$$

Returning to equation (1.1) and substituting from equation (1.19) gives the following relationship between the length of the interaction and m_a, which is as equation (1.6),

$$m_a = \frac{Jt_a}{2x_a} = \frac{\sqcap \omega_a}{c} = \frac{\sqcap}{\lambda_a} = \sqcap \kappa_a . \tag{1.22}$$

We find that for individual quantum mass, i.e. that of the neutron, electron, proton, muon, etc., x_q is equal to the Compton reduced wavelength, $ƛ_{C,q}$, for that quantum, as given by

$$x_q = ƛ_{C,q} = \frac{\sqcap}{m_q} . \tag{1.23}$$

Quantum analysis assumes the two fundamental invariants, $\hbar$ and c, to which we have now added an inertial constant, $\sqcap$. Some simple numerical analysis applied to the variables of mass, displacement, and time in conjunction with the equations for impulse, the inertial constant, interaction terminal velocity and action will help to clarify the geometric relationship of mass, length and time.

In the following table, Row A gives our initial, normalized condition for the variables valued in brackets in the left-hand column. In the remaining rows of this table we have simply substituted a new body of the given mass, and assumed different space and time values according to various impulse assumptions. The column on the right states whether the set of assumptions in the variables column violates any of the assumed invariants just stated. Rows B and C have the same v_f, but the space and time intervals differ and neither maintains the velocity of the initial condition. Row D maintains that velocity, but violates the action and the related inertial constant condition. It also departs from the initial value of the impulse. The stipulation of a set value for the impulse was a convenience for purposes of development of our argument, but it is not a necessary or even anticipated condition of a physical interaction. Row E is constrained to maintain that impulse, thus maintaining the velocity found in B and C, but results in a violation of all three invariants and is not a quantum solution.

Only Row F and the related G, while necessarily departing from the initial impulse, avoid a violation of the three invariants. What F and G show at a glance, assuming a quantum context, is that quantum inertial test mass is an inverse measure of space and time, the latter two of which are identically gauged in keeping with the development of the previous section on kinematics in which we saw that $r_0^o = t_0^o = R_0^o$.

In Row F, if the space and time standards are assumed to be smaller by the inverse of the factor γ due to a contraction in a moving frame after impulse, the increase in the mass is found to be by that same factor, showing that Row F is consistent with the postulates of special relativity. Row G, on the other hand, shows an increase in the space and time standards in keeping with a change in γ and a corresponding decrease in mass as we might find in a moving frame that has decreased its velocity from a prior greater differential with respect to some rest frame.

As a source, mass is a direct measure of the impulse as shown by the second column of Rows F and G. Further, since the space and time intervals are identical, and we might assume symmetrical, i.e. interchangeable, it is apparent that the impulse has the same dimensional form as the spin energy of the quantum. Again, using the angular frequency in computing the final velocity, we have the same form for the inertial constant and that velocity, so that in natural units,

$$c^2 = \text{ת}c = \hbar \tag{1.24}$$

and mass and frequency measure the same physical condition, interaction per time interval, i.e. the smaller the interaction time, the greater the mass and frequency, as

$$\frac{m}{\omega} = \frac{\hbar}{c^2} = \frac{\text{ת}}{c} = 1. \tag{1.25}$$

	$F(m,x_f,t_f) = mx_f t_f^{-2}$	$\int_{ti}^{tf} F(t)\,dt = Ft_f = J$	$\int_{ti}^{tf} J(t)\,dt = mx_f = \text{ת}$	$v_f = \frac{x_f}{t_f} = x_f\omega_f$	$\int_{xi}^{xf} J(x)\,dx = \text{ת}v_f = S(=\hbar)$	Violations of $\text{ת},v,S$
A	$(1,1,1)$	$\frac{1(1)}{1^2}(1)=1$	$1(1)=1$	$1 = 1(1)$	$1(1)=1$	
B	$(2,\frac{1}{2},1)$	$\frac{2(\frac{1}{2})}{1^2}(1)=1$	$1(1)=1$	$\frac{1}{2} = \frac{1}{2}(1)$	$1(\frac{1}{2})=\frac{1}{2}$	v,S
C	$(2,1,2)$	$\frac{2(1)}{2^2}(2)=1$	$1(2)=2$	$\frac{1}{2} = 1(\frac{1}{2})$	$2(\frac{1}{2})=1$	$\text{ת},v$
D	$(2,1,1)$	$\frac{2(1)}{1^2}(1)=2$	$2(1)=2$	$1 = 1(1)$	$2(1)=2$	$\text{ת},S$
E	$(2,\frac{1}{\sqrt{2}},\sqrt{2})$	$\frac{2(\frac{1}{\sqrt{2}})}{\sqrt{2}^2}\sqrt{2}=1$	$1(\sqrt{2})=\sqrt{2}$	$\frac{1}{2} = \frac{1}{\sqrt{2}}(\frac{1}{\sqrt{2}})$	$2(1)=2$	$\text{ת},v,S$
F	$(2,\frac{1}{2},\frac{1}{2})$	$\frac{2(\frac{1}{2})}{\frac{1}{2}^2}(\frac{1}{2})=2$	$2(\frac{1}{2})=1$	$1 = \frac{1}{2}(2)$	$1(1)=1$	None
G	$(\frac{1}{2},2,2)$	$\frac{\frac{1}{2}(2)}{2^2}(2)=\frac{1}{2}$	$\frac{1}{2}(2)=1$	$1 = 2(\frac{1}{2})$	$1(1)=1$	None

Table 1 – Numerical Analysis of Invariant Violation of Certain Variable Assumptions

The symmetries are yet more pronounced since the speed of light, written in terms of the properties of a wave, can be stated as the ratio of the angular frequency and angular wave number, κ,

$$c = \frac{\omega}{\kappa} \tag{1.26}$$

which when substituted into equation (1.25) gives us the symmetrical statement for the inertial constant,

$$\text{ת} = \frac{m}{\kappa}. \tag{1.27}$$

A couple of words are in order concerning the frequency, which are no doubt obvious to most readers. First, since it is an expression of the ratio between a count of the number of units or radian contemporaneous with a unit of time, in keeping with the comments concerning equation (0.1), it is equal to a count of one radian per fraction of some larger unit of time. A base or unit frequency would be an extremal, normalized frequency of one radian or other briefest instance of change per one smallest standard of time, $t_0^o = r_0^o = R_0^o$. Thus an interaction of the greatest frequency and therefore greatest energy per equation (1.20) will be the one of shortest duration. Second, while such normalized frequency might be a conventional angular frequency of one radian per unit of time, it might equally be one unit of space per unit of time as

$$c = r_0\omega_0 = r_0\frac{1}{t_0} = \frac{r_0}{t_0}. \tag{1.28}$$

If we state with respect to Rows F and G that

$$x_f = r_0 \tag{1.29}$$

then the displacement x_f resulting from impulse J, might be a reference to a rotational tangent vector at the circumference of the previously depicted rotating clock, instead of the customary translational displacement vector at or from a point-like particle. Such impulse, under the constraints of an invariant c, results in a contraction of the clock, and a decrease in r_0 and t_0 in keeping with γ, and mass is correspondingly measured as increased. Such impulse could be the result of an inelastic collision with a photon-like source or the acceleration arising from some field potential. It is important to note in regards to this last condition, that the increase in mass, as with the impulse, could be continual and not in discrete steps and still adhere to equation (1.20), since the frequency can increase continually, while the action, $S = \hbar$, remains invariant at any frequency.

To make graphic sense of this in terms of an inertial spacetime continuum, we can think of the aforementioned collision between A and a as an elastic, but constrained collision between two bodies of equal mass, so that their motion oscillates in simple harmonic motion. Next we consider a, instead of a body or particle, to be a 3 dimensional continuum, non-particulate in composition, isotropic but for a boundary plane at the point of impact from A, which is moving normal to and in the direction of a. Instead of the mass quantity of body a, continuum a has a linear inertial density. That density is variable and elastic in addition to being inertial, so that as A meets a, the inertial density immediately in front of the line of travel of A, i.e. the mass of a, increases, slowing A's velocity, and the continuum around the area of impact of A on a is strained and curved inward.

If we had assumed an <u>inelastic</u> collision, at some point the momentum of A would be absorbed by a, which would then remain in a distorted condition, marked by a finite degree of strain and curvature of the continuum around the area of impact, and the impulse would continue indefinitely into the interior of a.

By analogy with the first interaction of bodies *a* and *A* above, given a fixed momentum of *A*, the greater the inertial density, and therefore the mass of *a*, the smaller will be the penetration of *A* and the radius of the strain at the area of the impact. We can envision that there are two instances of curving, one as a generally deformed circular area around the area of interaction of *a* and *A* and the other along the sides of the generally conical deformation of the initial plane of the interaction. For simplicity we will assume that the radii are of equal magnitude, though necessarily of different sense.

With an elastic collision, at some point all the momentum of *A* will be transferred to *a*, but in the case of a continuum *a*, at such point all the kinetic energy of *A* is transferred to the elastic potential energy of the stress and strain at the deformation of *a*. As the shear forces in the plane of the interaction of *aA* exceed the compression force of *A* on *a*, a force which is transverse to the interaction plane, *a* will rebound and begin to work on *A*, which will travel in the opposite direction, eventually to be expelled from the plane of the initial impact. We imagine this interaction mirrored by a set up, Aa', opposite *a*, so that the total system aAa' constitutes a condition of simple harmonic motion.

Finally we can dispense with *A*, joining *a* to the other continuum, a', at the boundary plane, so that it is the aa' interaction that constitutes a resonant oscillation of a localized, spheroidal section of the combined continuum, in which $m_a = m_{a'} = m_0$.

Using equation (1.6), we can state the linear inertial density, λ_0, of the continuum at the system as follows,

$$\lambda_0 = \frac{m_0}{r_0} = \frac{\text{ת}}{r_0^2} = \text{ת}\kappa_0^2 . \tag{1.30}$$

This indicates that the linear inertial density is equal to the inertial constant times the curvature of the deformation or strain, as shown in the last term. Assuming an isotropic Gaussian curvature, *k*, given by

$$k = \kappa_0^2 = \frac{1}{r_0^2} , \tag{1.31}$$

this means that mass is a measure of linear curvature, given by the angular wave number, κ, once again indirectly related to the length scale, as

$$m_0 = \frac{\text{ת}}{r_0} = \text{ת}\kappa_0 . \tag{1.32}$$

Now we might stipulate that instead of a linear oscillation, this interaction forms a rotational oscillation in a plane normal to the intersection of *a* and a', along with an oscillation in that plane of intersection. Such oscillation would mimic the rotation of our three dimensional clock developed in the previous section on kinematics. It is suggested that the reader review that motion now.

Thus a geometrization of massive-particle mass involves the representation of such quanta as three dimensional clocks and indicates that particle mass is a measure of the frequency of the clock. As the above continuum is a representation of 3-D space, its oscillation represents an oscillation of spacetime. If that oscillation is seen to be at resonant frequency, ω_0, then we have the following relationship to the wave speed in such continuum

$$\kappa_0 = \frac{\omega_0}{c} \tag{1.33}$$

which when substituted into equation (1.30) gives the following wave equation, where τ_0 is the tension force in the continuum,

$$\lambda_0 = \frac{\sqcap\omega_0^2}{c^2} = \frac{1}{c^2}\tau_0 \tag{1.34}$$

From this discussion we can state some basic quantum dynamic properties of interest in terms of the inertial constant.

Interaction impulse = transverse wave momentum	$\Delta p = mc = \sqcap\omega$
Force – stress force = transverse wave force	$\tau = \sqcap\omega^2$
Action = spin angular momentum	$S = \hbar = \sqcap c = \sqcap\dfrac{\omega}{\kappa}$
Rest Mass	$m = \sqcap\dfrac{\omega}{c} = \sqcap\kappa$
Spin Energy	$E = mc^2 = \hbar\omega = \sqcap c\omega$

Lastly, the serendipity of this scenario, with the orthogonal, rotational transformation of a 3-D clock analogy for the mass oscillation of spacetime and its etymological symmetry with the orthogonal folding and rotation of dough in the kneading process of *massieren* is inescapable.

Gravitational Mass (Source)

In general relativity, gravitational source mass is converted from conventional units related to a force, M_{kg}, to units of length, M_l, by the conversion factor of G_N/c^2, where G_N is Newton's gravitational constant and c^2 is the speed of light in a vacuum squared, both of which are taken as free parameters, as

$$r = M_l = \frac{G_N}{c^2}M_{kg} = \left(7.424x10^{-28}\,\tfrac{m}{kg}\right)M_{kg}\,. \tag{1.35}$$

This procedure facilitates computation, as when used in a metric, so that if M_l is the geometrized mass of an extreme Kerr black hole, the reduced circumference at the horizon is $r_h = M_l$.

It bears noting that the relationship between the two measures of mass is <u>direct</u> and appears to be classical, or continuous, so that we can state a differential form

$$dr = dM_l = \frac{G_N}{c^2}dM_{kg} \tag{1.36}$$

We should acknowledge, however, the obvious and logical possibility that M_{kg} is an aggregation of some basic quantized units of mass of one or more magnitudes. Consideration of this equation using the smallest of rest-mass quanta, the electron, gives a linear measure of its mass in orders of magnitude of 10^{-58} meters. For the proton and neutron, the figure is slightly larger at 10^{-54} meters. However, all of these are much smaller than the Planck length of 10^{-35} the reputed smallest of determinable physical scales, raising possible theoretical questions about the use of equation (1.35) in determining a geometrized mass for individual quanta.

As previously discussed, from quantum theory, for individual rest-mass quanta, the closest equivalent to a length measure of mass is the quantum wavelength, albeit as an inverse measure; the Compton wavelength for a quantum at relative rest and the deBroglie wavelength for a quantum at relativistic velocity. For the most part we will confine ourselves to the reduced Compton wavelength of a quantum, *q*, or $\lambdabar_{C,q}$, and we will exclude from discussion photons or other energy quanta.

Once more, the mass of an individual quantum, a neutron, proton, electron, tau, or muon is related to its reduced Compton wavelength by the following, where the r_q is the reduced circumference and the norm of a polar coordinate system centered on *q*.

$$\lambdabar_{C,q} = \frac{\hbar}{c}\frac{1}{m_q} = \frac{\text{ת}}{m_q} = r_q\,. \tag{1.37}$$

In the SI system, ת (tav), evaluates as

$$\text{ת} \equiv \frac{\hbar}{c} = m_q r_q = 3.5176x10^{-43}\,kg\cdot m \tag{1.38}$$

Once again, in contrast to the direct relationship in the geometrization of mass in the classical application of general relativity, in quantum theory conventional mass is <u>indirectly</u> related to length. If we consider a relativistic quantum qualitatively, we know that the deBroglie wavelength decreases as the relativistic mass, and the particles momentum, increases, indicating once again the inverse relationship of mass and length.

2 – Derivation of Newton's Gravitational Law from Quantum and General Relativistic Principles

We would like to derive Newton's Gravitational Law from quantum principles, while in keeping with the principles of general relativity. The quantum principle we are interested in is the fundamental principle of a fundamental discrete unit or quantity. This means that we seek to express the gravitational force, F, of his law as a product of 1) the number, n_a, of some as yet unknown fundamental discrete units of mass, m_0, in two aggregate bodies of mass, M_a, 2) the curvature of space, k, expressed as the inverse square of the massive bodies separation in numbers of some as yet unknown minimum unit of length, specifically of a reduced circumference, r_0, and 3) a fundamental discrete unit or quantum differential of gravitational force, dG_0, as

$$F_{m_1 m_2 k} = n_{M1} n_{M2} n_r^{-2} dG_0 \tag{2.1}$$

We state Newton's Law, where G_N is Newton's gravitational constant, conventionally considered a free parameter, as

$$F_{M_1 M_2 k} = \frac{M_1 M_2}{R^2} G_N = M_1 M_2 k G_N \,. \tag{2.2}$$

Assuming a 3-space that is isotropic with respect to a source mass, M_1, here we have made use of the observation that the inverse square component of the distance of separation, R, of M_1 and M_2 is the reduced circumference of the spacetime around M_1, making the inverse of the square of R the measure of the Gaussian curvature, k, at the location of M_2, using

$$k = \frac{1}{R^2} = \kappa^2 \,. \tag{2.3}$$

It is also observed that with respect to a potential orbit of M_2 about M_1 this curvature and the inverse square component is equal to the square of the angular wave number, κ, of that orbit.

It is obvious that the left hand side of equation (2.2) represents a force. Some reflection will tell us that if it is to be related to general relativity, the right hand side must represent the product of a 4-stress, T, and an area, A, or in keeping with the last paragraph, an inverse curvature. Thus this equation is dimensionally equivalent to

$$F = T\mathrm{A} = Tk^{-1} \,. \tag{2.4}$$

Some rearrangement gives us a scalar form

$$k = F^{-1} T \tag{2.5}$$

where the curvature of spacetime given by the left term is related to the stress-energy density of the right by the inverse force. This is thus related to the field equation of general relativity, customarily expressed in tensor form as

$$G_{\alpha\beta} = -8\pi G_N T_{\alpha\beta} \tag{2.6}$$

where the Einstein curvature tensor on the left is so related to the stress-energy tensor on the right by the geometrically based numerical coefficient and Newton's gravitational constant, which we will see conceals a force differential.

Analyzing G_N dimensionally, we know it has the dimensions of distance, r, cubed divided by the product of a mass, m, and time, t, squared. If it in fact conceals a force differential, extracting that force in the third term shows G_N to be the product of that force and the inverse square of a linear inertial density, λ, as

$$G_N = \frac{r^3}{mt^2} = \frac{r^2}{m^2}\frac{mr}{t^2} = \lambda^{-2} dF \tag{2.7}$$

The inertial densities, then convert the mass on the left side of Einstein's field equation, in equation (2.6), of which there are two, one in Newton's constant and one in the stress tensor, to the product of two distances. This takes us half the way to equation (2.5), which has an inverse force on the right while the G_N in Einstein's equation has a direct force. Using the fundamental identity of space and time as shown in equation (0.7), we can make the following dimensional substitutions into equation (2.7),

$$G_N = \frac{r^3}{mt^2} = \frac{(it)^3}{m(-ir)t} = \frac{t^2}{mr} = dF^{-1} \tag{2.8}$$

which converts G_N to an inverse force and equation (2.6) assumes the dimensional form of equation (2.5).

Expressed as a force, gravity is centrally directed toward the bodies of mass and within the context of a flat spacetime, assumed to be isotropic about each. The curvature in such conditions is considered generally spherical, so that some rearrangement of equation (2.4) in the absence of any rotation of the two bodies about each other, results in an isotropic dimensional expression of tension stress, f_3, where the subscript indicates the dimensional order of the stress

$$f_3 = \frac{F}{\mathrm{A}} = Fk\,. \tag{2.9}$$

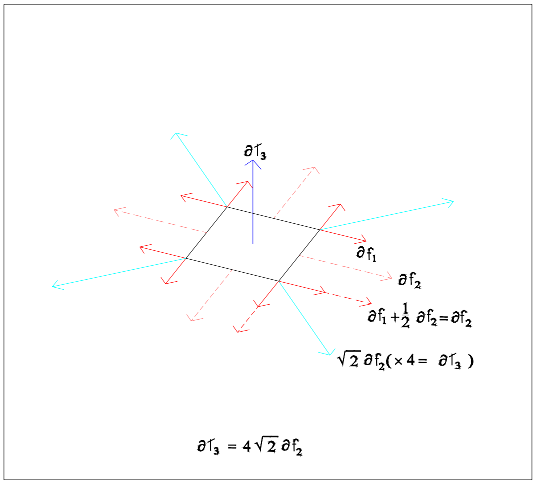
The stress in the case of general relativity is a 4-stress, however, so that we are looking for a formulation that makes explicit the general relationship

$$f_3 \equiv \frac{F}{A} \sim T \equiv T_4\,. \tag{2.10}$$

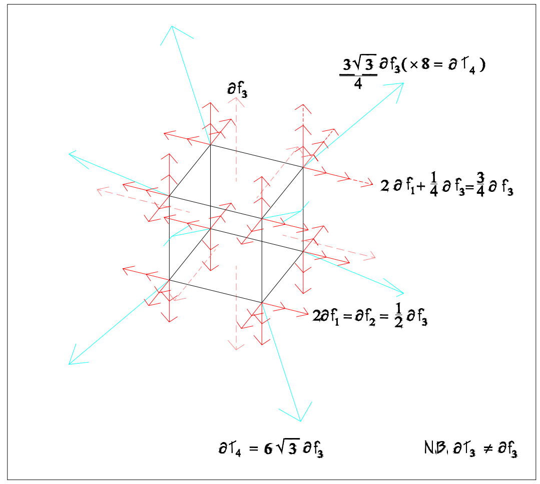
Additionally, we are interested in the 4-stress associated with an accelerating expansion of space, so we take a closer look at the geometry of stress, specifically of isotropic expansion stress. We examine the case of energy density - stress in an n-manifold that is expanding in response to its expanding n+1-core. First, in Stress Diagram 1 we examine the differential area of a 2-sphere on a 3-ball, such as an expanding balloon. We imagine that the balloon is expanding due to a differential pressure normal to the balloon surface, so that there is a 3-stress (tension), dT_3, orthogonal to the balloon's surface, the 2-sphere. We look at a differential square on the surface of the balloon and see that the sum of the 2-stress (transverse), df_2, in the balloon surface at that locus should be equal to the orthogonal tension stress, or

$$dT_3 = \gamma_2 df_2 \tag{2.11}$$

where γ_2 is a geometric factor summing the transverse stress.



Stress Diagram 1



Stress Diagram 2

It is the displacement of the vertices of the square that defines the change, so instead of a normal unit vector to each mid-edge of the differential square, we stipulate a ½ vector at each vertex, along with a ½ shear vector, giving a total of 8, 1 vectors at the four vertices. With a total of 4 resultants of the vector pairs at each vertex, we have

$$\gamma_2 = 4\left(\sqrt{1^2 + 1^2}\right) = 4\sqrt{2} \tag{2.12}$$

In terms of elasticity theory, this is equivalent to an extremely high, ideally infinite bulk modulus and a Poisson's ratio of ½, and equation (2.11) becomes

$$dT_3 = 4\sqrt{2}df_2 \tag{2.13}$$

Extending this approach with analogous elasticity conditions to a 3-space on a 4-core, in Stress Diagram 2 we have a 4-stress, (which necessarily cannot be shown) normal to and equal to an isotropic 3-stress, as

$$dT_4 = \gamma_3 df_3 . \tag{2.14}$$

This time we consider a differential cube, and instead of the customary assignment of an orthonormal tension stress vector to the center of each of the faces of the cube, we assign a quarter of each normal vector to the corners of each face, thereby identifying them with the shear vectors of the two adjacent surfaces. This is equivalent to a Poisson's ratio of 1/3. The sum of these ¼ tension vectors and the two parallel ¼ shear vectors is a ¾ vector, so that there are 3, ¾ orthogonal stress vectors at each vertex. The resultant of the three orthogonal components at each corner then, aligned with the cubic diagonal, is the total stress contributed to each of the 8 vertices by an isotropic stress, so that the geometric factor relating the stresses in equation (2.14) is

$$\gamma_3 = 8\left(\sqrt{\left(\tfrac{3}{4}\right)^2 + \left(\tfrac{3}{4}\right)^2 + \left(\tfrac{3}{4}\right)^2}\right) = 8\frac{3\sqrt{3}}{4} = 6\sqrt{3} \tag{2.15}$$

and equation (2.14) becomes

$$dT_4 = 6\sqrt{3}df_3 \tag{2.16}$$

Next we examine a scalar expression of the equation (2.10) in light of this adjustment, where we specify that A_0 is a fundamental or quantum unit area,

$$\gamma_3^{-1} T = \frac{F}{A_0}, \tag{2.17}$$

with the derivatives for an invariant T being

$$\gamma_3^{-1} dT = \frac{\partial T}{\partial F} dF - \frac{\partial T}{\partial A} dA = \frac{1}{A_0} dF - \frac{F}{A_0^2} dA = 0\,. \tag{2.18}$$

Separating and inverting this function we have the two following differential equations, the first of which is straight forward,

$$dF = \gamma_3^{-1} A_0 dT \equiv \left(\gamma_3^{-1} \kappa_0^{-2} dT \right) \tag{2.19}$$

and the second one expressing various parsings of interest, especially those in which the stress force is removed from the equation,

$$\begin{aligned} dA &= -\gamma_3^{-1} \frac{A_0^2}{F} dT = -\frac{F}{\gamma_3^{-1} T^2} dT = -\frac{A_0}{T} dT \\ &= -A_0 d \ln T \equiv \left(-\kappa_0^{-2} d \ln T \right) \end{aligned} \tag{2.20}$$

According to the above specifications a quantum formulation for Newton's Law, as previously stated, would be

$$F_{m_1 m_2 k} = n_{M1} n_{M2} n_r^{-2} dG_0\,. \tag{2.21}$$

An aggregate mass is the product of the number of quanta in that aggregate times the fundamental unit of mass or with rearrangement

$$n_{Ma} = \frac{M_a}{m_0} \tag{2.22}$$

and the reduced circumference of the separation of the two bodies of mass is the product of the number of unit lengths in that separation and the minimum or quantum unit length, or

$$n_r = \frac{R}{r_0}\,. \tag{2.23}$$

Substituting equation (2.22) and equation (2.23) into equation (2.21) gives

$$F_{M_1 M_2 k} = \frac{M_1 M_2}{R^2} \left(\frac{r_0^2}{m_0^2} dG_0 \right) \tag{2.24}$$

Assuming that the gravitational quantum is equivalent to the formulation from equation (2.19) and substituting from its middle term, gives the following, in which the stress differential is normalized in its relationship to dG_0 as $dT_0 = 1$,

$$F_{M_1 M_2 k} = \frac{M_1 M_2}{R^2} \left(\gamma_3^{-1} \frac{r_0^4}{m_0^2} dT_0 \right) = \frac{M_1 M_2}{R^2} G_N\,. \tag{2.25}$$

Apparently, the bracketed term is equal to Newton's constant.

In keeping with quantum principles, we state the relationship between the above postulated quantum mass, m_0, and length, r_0, the latter stated as the reduced Compton wavelength,

$$m_0 = \frac{\hbar}{c} \bar{\lambda}_{C,0}^{-1} = \frac{\hbar}{c} r_0^{-1} = \frac{\text{ת}}{r_0}. \tag{2.26}$$

The ratio, λ_0, then is the linear inertial density of the quantum fundamental as

$$\lambda_0 = \frac{m_0}{r_0} = \frac{\text{ת}}{r_0^2}. \tag{2.27}$$

Here we restate ת (tav), the inertial constant, an invariant of a quantum interaction, as the integral of the interaction time interval, $t_f = t_f - t_i$, times the impulse $\boldsymbol{J}$, or

$$\text{ת} = \int_{t_i}^{t_f} \boldsymbol{J}(t)\, dt = \int_{t_i}^{t_f} \boldsymbol{F}(t)\, t_f \; dt = \tfrac{1}{2} \boldsymbol{F}\, t_f^2 = m\, \boldsymbol{x}_f\,. \tag{2.28}$$

where the impulse is defined as

$$\boldsymbol{J} = \int_{t_i}^{t_f} \boldsymbol{F}(t)\, dt = \Delta P \tag{2.29}$$

and

$$\boldsymbol{F} = m \; \frac{2\, \boldsymbol{x}_f}{t_f^2}\,. \tag{2.30}$$

Note again the relationship of ת to Maupertuis' principle and definition of the action, S, as

$$S = \int_{x_i}^{x_f} \boldsymbol{J}(x) \cdot d\, \boldsymbol{x} = \frac{2m}{t_f} \int_{x_i}^{x_f} \boldsymbol{x}_f \cdot d\, \boldsymbol{x} = m\, \boldsymbol{x}_f \cdot \frac{2\, \boldsymbol{x}_f}{2 t_f} = \text{ת} \cdot \frac{v_f}{2} = \text{ת} \cdot c = \hbar \tag{2.31}$$

Thus

$$\text{ת} \equiv \frac{\hbar}{c} = \frac{\hbar}{\omega_f x_f} = \hbar \frac{dt}{dx} \tag{2.32}$$

We substitute from equation (2.26) into the bracketed term of equation (2.25), and get

$$G_N = \frac{dG_0}{\lambda_0^2} = \frac{r_0^4}{\text{ת}^2} dG_0 = \gamma_3^{-1} \frac{r_0^6}{\text{ת}^2} dT_0\,, \tag{2.33}$$

which in a natural system simply equals γ_3^{-1}.

After some rearrangement, we have

$$r_0 = \left(\gamma_3 \text{ת}^2 G_N dT_0^{-1} \right)^{1/6}. \tag{2.34}$$

Since with respect to dG_0, dT_0 equals 1, and as we know the other invariants in the right hand term, we can solve for r_0, and find that in the SI system it equals the reduced Compton wavelength of the neutron or

$$r_{0n} = \bar{\lambda}_{C,n} = 2.10019 \ldots x 10^{-16} m \tag{2.35}$$

within the standard uncertainty for G_N. The "n" in the subscript "$0n$" is in some contexts redundant and simply emphasizes the neutron scale as the fundamental, quantum scale. All other values for the fundamental properties incorporate and can be computed from this value. Therefore, the fundamental gravitational mass is the neutron mass or

$$m_0 = m_n = 1.67492...x10^{-27}\,kg\,. \tag{2.36}$$

The gravitational quantum then is variously

$$dG_0 = \gamma_3^{-1}\kappa_{0n}^{-2}dT_0 = \gamma_3^{-1}\mathrm{A}_{0n}dT_0 = \gamma_3^{-1}T_0 d\mathrm{A} = 4.244...x10^{-33}\,N\,, \tag{2.37}$$

where the last algebraic term makes use of equation (2.20).

Some rearrangement gives

$$T_0 = \gamma_3\frac{dG_0}{d\mathrm{A}_0} = \frac{\mathrm{A}_{0n}}{d\mathrm{A}_0}dT_0 \tag{2.38}$$

With this development, we can get the spin energy density-stress, T_0, on the neutron, which we assume to be a quantum waveform, where E_{0n} is the spin energy of the neutron and

$$\tau_0 = \sqcap\omega_0^2 \tag{2.39}$$

is the transverse wave force of the oscillation,

$$\gamma_3^{-1}T_0 = \frac{E_{0n}}{r_{0n}^3} = \frac{m_n c^2}{r_{0n}^3} = \frac{\sqcap\omega_0^2}{r_{0n}^2} = \frac{\tau_0}{r_{0n}^2} = 1.625...x10^{37}\,N/m^2\,. \tag{2.40}$$

Substituting this into equation (2.37) for the gravitational quantum and rearranging, we get the following expression and value for the quantum or unit area differential, $d\mathrm{A}_0$, which we find is equal to the Planck area,

$$d\mathrm{A}_0 = \gamma_3 T_0^{-1}dG_0 = \mathrm{A}_{Pl} = 2.6116...x10^{-70}\,m^2 \tag{2.41}$$

This analysis indicates that Newton's gravitational constant contains a quantum differential, and that the neutron scale is the fundamental scale of an expanded spacetime. It also indicates a relationship to the Planck scale, and we would like to determine more of that relationship next.

3 – Analysis of the Relationship between the Neutron and the Planck Scale

If we use the conventional geometrization factor from general relativity for mass, G_N/c^2, for the neutron we get a length measure of a hypothetical quantum black hole horizon as

$$r_{hn} = m_{l,n} = \frac{G_N}{c^2} m_n = 1.243...x10^{-54}\,m \,. \tag{3.1}$$

Comparing this with the neutron reduced Compton, we get the dimensionless number

$$\frac{m_{l,n}}{\lambdabar_{C,n}} = \frac{r_{hn}}{\lambdabar_{C,n}} = 5.92...x10^{-39} \,. \tag{3.2}$$

Squaring equation (3.1) to get the inverse curvature of a hypothetical quantum inertial sink at that scale gives

$$r_{hn}^2 = 1.545...x10^{-108} \tag{3.3}$$

which is related to the Planck area by the same ratio or

$$\frac{r_{hn}^2}{\mathrm{A}_{Pl}} = 5.92...x10^{-39} \,. \tag{3.4}$$

It bears noting that this is in the range of the factor separating the gravitational and the strong interactions.

Using the structure for Newton's constant developed above, we analyze the conventional geometrization factor, where we make use of the classical wave relationship,

$$\tau_0 = \lambda_0 c^2 \tag{3.5}$$

in which τ_0 is the linear tension force and in this case the transverse wave force in a wave bearing medium, λ_0 is the linear inertial density of that medium and c is its speed of wave propagation. We find that conversion factor is equal to the differential of the natural log of the expansion stress divided by the linear inertial density,

$$\frac{G_N}{c^2} = \left(\frac{dG_0}{\lambda_0^2}\right)\frac{1}{c^2} = \frac{1}{\lambda_0}\left(\gamma_3^{-1}\frac{r_{0n}^2}{\lambda_0 c^2}\right)dT_0 = \frac{1}{\lambda_0}\left(\gamma_3^{-1}\frac{r_{0n}^2}{\tau_0}\right)dT_0 = \frac{1}{\lambda_0 T_0}dT_0 \tag{3.6}$$

$$\frac{G_N}{c^2} = \frac{1}{\lambda_0 T_0}dT_0 = \frac{d\ln T_0}{\lambda_0} = \frac{\lambdabar_{C,n}}{m_n}d\ln T_0 \tag{3.7}$$

Using CODATA values for the neutron mass and reduced Compton to determine λ_0, we can solve for $d\ln T_0$ and get the factor in equations (3.2) and (3.4)

$$d\ln T_0 = T_0^{-1}dT = \gamma_3^{-1}\frac{r_{0n}^3}{m_n c^2}dT = 5.92146...x10^{-39} \tag{3.8}$$

Inverting and multiplying through by $dT_0 = 1$ gives the value of T_0,

$$T_0 = \gamma_3\frac{m_n c^2}{\mathrm{A}_{0n} r_{0n}} = \gamma_3\frac{\lambda_0 c^2}{\mathrm{A}_{0n}} = 1.6888...x10^{38}\,N/m^2 \tag{3.9}$$

from which we can get the transverse quantum wave force of the neutron

$$\tau_{0n} = \gamma_3^{-1}T_0\mathrm{A}_{0n} = 7.1676...x10^{5}\,N \tag{3.10}$$

Assuming that the gravitational quantum is the differential of the quantum transverse wave force with respect to differential stress, $\tau'(T)$, we have the ratio of that differential and the wave force itself, $\tau(T)$, or equation (2.37) over equation (3.10)

$$d\ln T_0 = \frac{\tau'(T)}{\tau(T)} = \frac{d\tau}{\tau_{0n}} = \frac{dG}{\tau_{0n}} = 5.92146...x10^{-39} \tag{3.11}$$

which is the ratio of the gravitational and the strong interactions.

Rearranging equation (3.10) and taking the derivative of inverse curvature with respect to the isotropic stress results in an evaluation equal to the Planck area,

$$d\mathrm{A}_0 = -\gamma_3 \frac{\tau_{0n}}{T_0^2} dT_0 = -\mathrm{A}_{0n} d\ln T_0 = -\mathrm{A}_{Pl}\,. \tag{3.12}$$

once again indicating that the Planck area represents a differential of expansion stress. To verify this statement, we substitute equations (2.39), (3.5), and (3.9) for the expansion force and stress into the second term here and find

$$\begin{aligned} d\mathrm{A}_0 &= -\gamma_3 \frac{\sqcap\omega_{0n}^2}{\gamma_3^2 \lambda_0{}^2 c^4 \mathrm{A}_0^{-2}} dT_0 = \left(-\gamma_3^{-1} \frac{\mathrm{A}_0}{\lambda_0{}^2} dT_0\right) \frac{\sqcap\omega_{0n}^2 r_{0n}^2}{c^4} \\ &= -G_N \frac{\sqcap c^2}{c^4} = -G_N \frac{\hbar}{c^3} \end{aligned} \tag{3.13}$$

From this analysis of the differential nature of the Planck area and the endnote comments,[i] which suggest expansion along a hyperbolic manifold, from equation (3.12) we can show the Planck length as a differential value, as

$$dr_0 = \left|d\mathrm{A}_0\right|^{\frac{1}{2}} = r_{0n}\sqrt{d\ln T_0} = r_{Pl} = 1.6161...x10^{-35}\,m\,. \tag{3.14}$$

4 – Cosmological Implications

Basic to our discussion is the assumption that spacetime is expanding relative to our local frame of reference. This means that over time a local fixed unit length standard becomes an ever decreasing proportion of some linear measure of the cosmic extent. If we project backwards in time, we can assume that at some point that measure was potentially equal to the current local length standard or unity.

The current concept of a big bang start of cosmic spacetime expansion implies an initial condition of maximum inertial density, possibly infinite, which decreases with the expansion of space from an extremely small volume, possibly zero, i.e. from a singularity. This begs the question of what triggered the release of the attendant pent up stress such initial inertial density represents. Instead of emergence from a singularity, the space component of spacetime can be modeled as a boundary on the next higher dimensional manifold itself under expansion, analogous to a circle drawn on the surface of an expanding balloon. Alternately, we might imagine a spherical balloon of fixed size with a circular wave emanating from one spot, widening in diameter as it approaches an equator before shrinking again as it nears the antipode. An analogous inertial spacetime oscillates on a cosmic scale between a maximum density and rarification, between a maximum compression and maximum extension. The fact that the expansion appears to be accelerating indicates that the expansion rate is best understood exponentially. We can then take the condition of maximum density as unity instead of as a singularity, and gauge any expansion with respect to that unity for A_0 and r_0 as inversely related to the associated increase in stress T_0 due to expansion according to equations (3.12) and (3.14).

The current expansion factor, $\kappa_{\exp}$, then is the ratio of the current fundamental scale, the neutron scale, to the Planck length is equal to the inverse square root of the differential natural log of the expansion stress,

$$\kappa_{\exp} = \frac{r_{0n}}{dr_0} = \sqrt{d \ln T_0}^{-1} = 1.29952...x10^{19} \tag{4.1}$$

As this expansion is at an exponential rate, in terms of doubling from an initial condition of maximum density equal to the linear inertial density of the neutron scale, λ_0, with time and space normalized, in terms of the whole or an arbitrary unit standard, cosmic expansion, C_x, is

$$C_x = \ln 2\left(\kappa_{\exp}\right) = 9.00764...x10^{18} \text{ light seconds } = 2.8544...x10^{11} \text{ light years} \tag{4.2}$$

Note that the last term would indicate, if interpreted as a straight line increase at the speed of light, an expansion age of the cosmos of 285.44 billion years.

An exponential expansion rate, X_e, derived in the full development of this model and shown to equate to a predicted Hubble rate of 72.791 km/mps/sec, shows the change in unit scale per second as

$$X_e = H_0 = \frac{\Delta r_0}{r_0} \Big/ \text{second} = 2.35896...x10^{-18} / \text{ s} \tag{4.3}$$

If we interpret this as a straight line expansion rate from an initial singularity, inverting would give the age of the cosmos in current units as

$$X_e^{-1} = 13.433 \text{ billion years} \tag{4.4}$$

However, if the Hubble rate is exponential or compounding, the following gives the Hubble time, τ_H, as a time in current units for a doubling in spatial linear extent, or

$$\tau_H = \ln 2 X_e^{-1} = 9.311 \text{ billion years} \tag{4.5}$$

The product of the expansion rate and the expansion factor is the number of doublings or

$$X_e \kappa_{\exp} = 30.655\ldots \text{ doublings } = 285.43 \text{ billion years} \tag{4.6}$$

Following this logic, if the wavelength of the cosmic microwave background is approximately 3.3*mm* and indicates an expansion along with spacetime from a primal epoch of beta decay as gauged by the electron Compton wavelength, dividing the natural log of such expansion by the natural log of 2 would give the number of doublings based on those parameters or

$$\ln\left(\frac{.0033}{\lambda_{C,e}}\right)(\ln 2)^{-1} = \frac{\ln 1.360\ldots x10^9}{\ln 2} = 30.34\ldots \text{ doublings } = 282.5 \text{ billion years} \tag{4.7}$$

in very close agreement with equation (4.6).

This observation indicates that r_0 remains stable as spacetime and the CMB expands and indicates that such quanta did not have a geometry of the Planck scale at an early epoch, which instead of starting from a singularity with all the physical dilemma that entails, started expansion from a maximum finite density. The Planck length, then, is the ratio of the neutron reduced Compton and the cosmic extension from an initial compact condition of maximum density, and continues to decrease with expansion.

Alternately, but not contradictory, if we think of the cosmic extent of 3-space as a fixed unit, what appears mathematically from a local perspective as expansion is from the universal perspective a process of regional and local concentration of inertial density. With respect to our analogy of the fixed balloon above, the linear (and area) density of the balloon in the absence of a wave is invariant over the surface of the sphere, but a wave moving over its surface creates a density differential at the wave front, increasing as it approaches a pole and antipole and decreasing as it approaches an equator. From the reference frame of the traveling wave front approaching the poles, the stress related to the wave front, T_0, increases and r_0, as a related unit standard which in the case of the balloon we might give as the distance perpendicular to the given polar diameter, decreases over time. The ratio of r_0 with respect to the balloon's extent, B_x, its radius at the equator, represents a decreasing differential length, dr_0, and can be expressed as the cosine of the angle of declination of the wave front.

The wave front in this analogy represents the current local quantum scale given by r_{0n}. If we were to rotate the balloon about the given polar axis at the same frequency as the wave's movement over its face, each point in the wave front would mimic the action of our 3-D clock. From either of the above perspectives, the energy per cosmic extent is invariant and cosmological red-shift is apparent, and in neither case is the Planck scale a fixed discrete scale.

Black Hole Metrics

Assuming that the above and supportive analysis does indicate that the neutron is a quantum inertial sink, but not a quantum black "hole", then a maximum linear inertial density is given by

$$\lambda_0 = \frac{m_n}{r_{0n}} = 7.975...x10^{-12}\, kg/m \tag{4.8}$$

This would seem quite small, but for its bulk implications. For a volume density, we would figure the number of hypothetical fundamental rest mass quanta per volume of such quanta, tightly packed. Using a packing system of one sphere with twelve contacting identical spheres, and disregarding any expansive effects of spin, charge, etc., we can compute the theoretical maximum density and find that it equals roughly

$$2.2549...x10^{46}\, quanta \;/\; m^3 \tag{4.9}$$

Inverting the neutron mass gives the number of such quanta per kilogram or

$$5.9704...x10^{26}\, quanta \;/\; kg \tag{4.10}$$

for a maximum theoretical density of

$$3.7768...x10^{19}\, kg \;/\; m^3 \tag{4.11}$$

or a density per sphere of one meter radius

$$\rho_{sphere} = 1.5820...x10^{20}\, kg \;/\; meter\ sphere \tag{4.12}$$

From this we can find a threshold black hole mass, $M_{kg,TBH}$ for an aggregation of quanta by using the following for a flat Euclidean space, where r_{Max} is the reduced circumference of a celestial body of maximum density,

$$r_{Max} = \left(\frac{M_{kg,TBH}}{\rho_{sphere}}\right)^{\frac{1}{3}} \tag{4.13}$$

Assuming $r_h = M_l$ as with an extreme Kerr spacetime

$$\frac{G_N}{c^2} M_{kg,TBH} = M_l = r_h = r_{Max} = \left(\frac{M_{kg,TBH}}{\rho_{sphere}}\right)^{\frac{1}{3}} \tag{4.14}$$

for an extreme Kerr horizon gives

$$M_{kg,TBH} = \left(\frac{c^6}{G_N{}^3 \rho_{sphere}}\right)^{\frac{1}{2}} = 3.930x10^{30}\, kg \tag{4.15}$$

which using the above density gives us the evaluations in the following table or approximately two solar masses for the threshold.

Here in column 3, from equation (4.13) we compute the r_{Max} for various celestial bodies, Earth, Sun, Milky Way galactic BH and Virgo cluster BH, and include the theoretical threshold size black hole and the Universe, as listed in column 1. "Flat Spacetime" does not specify that the pertinent body has no curvature effect on the surrounding spacetime, but rather that the curvature of individual quanta, i.e. quantum gravity, is not effected by the aggregate mass and remains the same as for an individual quantum in isolation in flat spacetime, i.e. there is no assumed collapse of each quantum waveform toward a quantum singularity, though there may be a state similar to a Bose-Einstein condensate. The fourth column gives the reduced circumference at the horizon of an extreme, charge free, Kerr black hole according to the conventional interpretation of general relativity. The fifth column simply makes explicit whether the third column figure resides within the fourth. This indicates that the rest mass quanta inside a black hole horizon could congregate at maximum density without precipitating a singularity.

	Mass in kg	Radius, r_{Max} in m, Density = ρ_{Sphere} Flat Spacetime	Mass in meters $M_l = \frac{G_N}{c^2} M_{kg} = r_h$	Is r within $M_l = r_h$ at Horizon?
Earth	$*5.9742x10^{24}$	33.55	$4.44x10^{-3}$	No
Sun	$*1.989x10^{30}$	$2.325x10^{3}$	$1.477x10^{3}$	No
Kerr BH threshold	$3.930x10^{30}$	$2.913x10^{3}$	$2.913x10^{3}$	At Horizon
Milky Way	$*5.2x10^{36}$	$3.20x10^{5}$	$3.86x10^{9}$	Yes
Virgo cluster	$*6x10^{39}$	$3.36x10^{6}$	$4.45x10^{12}$	Yes
Universe	$1.67x10^{53} = 10^{80}\,nucleon$	$1.02x10^{11}$	$1.24x10^{26} = 13.1x10^{9}$ light yrs	Yes

*Figues from Exploring Black Holes, by Taylor and Wheeler, Addison Wesley Longman, 2000

Chart of Various Celestial Mass Geometrizations

Of interest is the fact that the universe appears to be within its own horizon, which conventionally would tend to imply that its constituents should be contracting, and that there are black holes within black holes. Also the mass in meters being equal to the reputed age of the universe times the speed of light seems a bit serendipitous unless of course that mass, i.e. the number of currently theorized nucleons, was estimated using the above geometrization equation. But this figure is not the currently theorized (observed) extent of the universe, which is in the 150 billion light year range. Finally it is noted that the hypothetical mass of the known universe at maximum density and a radius of 102 million kilometers, would fit inside the earth's solar orbit in flat spacetime.

5 – The Quantum Metric

We turn now to the metric, specifically a chargeless extreme Kerr metric in the equatorial plane (the ϕ coordinates are suppressed), in which the angular momentum parameter, *a*, is equal to the horizon reduced circumference and the geometrized mass, or $a = r_h = M_l$.
The timelike metric at the horizon is

$$d\tau^2 = \left(1 - \frac{2M_l}{r_h}\right)dt^2 + \frac{4M_l a}{r_h}dtd\theta - \frac{dr^2}{\left(1 - \frac{2M_l}{r_h} + \frac{a^2}{r_h^{\ 2}}\right)} - \left(1 + \frac{a^2}{r_h^{\ 2}} + \frac{2M_l a^2}{r_h^{\ 3}}\right)r_h^{\ 2}d\theta^2 \quad (5.1)$$

Substituting for $a = M_l$ gives

$$d\tau^2 = \left(1 - \frac{2M_l}{r_h}\right)dt^2 + \frac{4M_l^2}{r_h}dtd\theta - \frac{dr^2}{\left(1 - \frac{M_l}{r_h}\right)^2} - \left(r_h^{\ 2} + M_l^2 + \frac{2M_l^3}{r_h}\right)d\theta^2 \quad (5.2)$$

We make the following observation concerning the dr^2 term. While the conventional interpretation is that the term "blows up" as the denominator approaches zero, and any infalling test particle transits the horizon, the math can also be interpreted in terms of a limit for radial motion. A mathematical slight-of-hand is at work in the formulation, since the differentials are deemed to approach zero in the limit, but are effectively treated as dimensional units equal to one. This is necessary since the product of the co-efficients and zero would be zero, and is warranted since we find a similar differential on the left side of the equation. It does not, however, address the situation if the metric component represented by the differential has a natural limit as does the radial component of a polar coordinate system.

Thus if the horizon in an extreme Kerr spacetime represents a limit, *dr* equals zero at the limit of that horizon coincident with the term in the denominator, the coefficient and the differential cancel and the result is simply -1 as shown below. The horizon, then, is effectively a physical asymptote. Thus at the event horizon, where $r = r_h = M_l$ this simplifies to

$$d\tau^2 = -dt^2 + 4r_h dtd\theta - (2r)^2\,d\theta^2 - dr^2 = (idt - i2r_h d\theta)^2 + (idr)^2 \quad (5.3)$$

This can be factored as a complex number and its conjugate

$$d\tau^2 = \left[(idt - i2r_h d\theta) + i(idr)\right]\left[(idt - i2r_h d\theta) - i(idr)\right] \quad (5.4)$$

or can be simplified as follows,

$$d\tau^2 = \left[(idt - i2r_h d\theta) - dr_h\right]\left[(idt - i2r_h d\theta) + dr_h\right] \quad (5.5)$$

where r_h is the reduced circumference at the horizon and $dr_h = 0$ is a zero vector with respect to the radial, giving a proper time of

$$d\tau = \pm i(dt - 2r_h d\theta) \quad (5.6)$$

If we assume that for bookkeeper time the differential is in the plane of the horizon, and time flows with the rotational motion of the ergosphere, so that

$$dt = r_h d\theta \tag{5.7}$$

then the proper time is found to flow orthogonally to that rotational motion, into the negative and positive ϕ coordinates, since

$$d\tau = \mp i dt \tag{5.8}$$

This will be significant in our statement of the quantum metric.

From this perspective, at the static limit and the start of the ergosphere, where $r = 2M_l$, pure radial motion is no longer possible, and a rotational component or frame dragging element is injected into the equation so that at the event horizon, all motion is rotational as indicated by the "imaginary" or orthogonal senses. Instead of gravitational collapse, this argues that any incremental matter or light accruing to the inertial sink is smeared out and bound at the horizon.

We now get to the meat of the matter with an expression of the quantum metric. The dynamics of the quantum waveform is not extremely complicated, but it does involve some rather lengthy, non-standard analysis using methods of complex classical wave physics extended to 4 dimensions, and is beyond the scope of the present discussion. We will simply state that its kinematics prevent the orientation entanglement condition.

The timelike quantum metric is given as a modified chargeless extreme Kerr metric. The modification is in the ϕ coordinates as shown here, where the quantum mass has been explicitly geometrized,

$$d\tau^2 = \left(1 - \frac{2r_{0n}}{r_{0n}}\right)dt^2 + \frac{4r_{0n}{}^2}{r_{0n}} dt d\theta - \frac{dr^2}{\left(1 - \frac{r_{0n}}{r_{0n}}\right)^2} - R^2 d\theta^2 \mp \left\{\left(e^{\pm i(\omega_0 t \mp \theta)} L d\phi\right)^2\right\} \tag{5.9}$$

The caveat stated earlier concerning the limit of radial motion represented by r_{0n} remains. In the last term, the complex exponential is defined as

$$\begin{aligned} e^{\pm i(\omega_0 t \mp \theta)} &= \mathrm{Re}\left(e^{\pm i(\omega_0 t \mp \theta)}\right) = \cos_{ccw}\left(\omega_0 t + \theta\right) \text{ or } \cos_{cw}\left(\omega_0 t + \theta\right) \\ &= \cos\left(\omega_0\left(+t\right) - \theta\right) \text{or } \cos\left(\omega_0\left(-t\right) + \theta\right) \end{aligned} \tag{5.10}$$

Either the real or the imaginary part could of course be used. The *ccw* term indicates rotation in the upper hemisphere according to the right hand rule, while the *cw* term indicates clockwise rotation in the bottom hemisphere according to the left hand rule, <u>when viewed from the exterior of the corresponding rotational pole</u>.

The plus and minus curly brackets has the following definition and indicates a flipping of the sign of the $d\phi$ vector, with every π rotation of θ, plus being parallel and minus being anti-parallel with respect to the spin axial vector. It thus performs a function similar to a mathematical spin matrix.

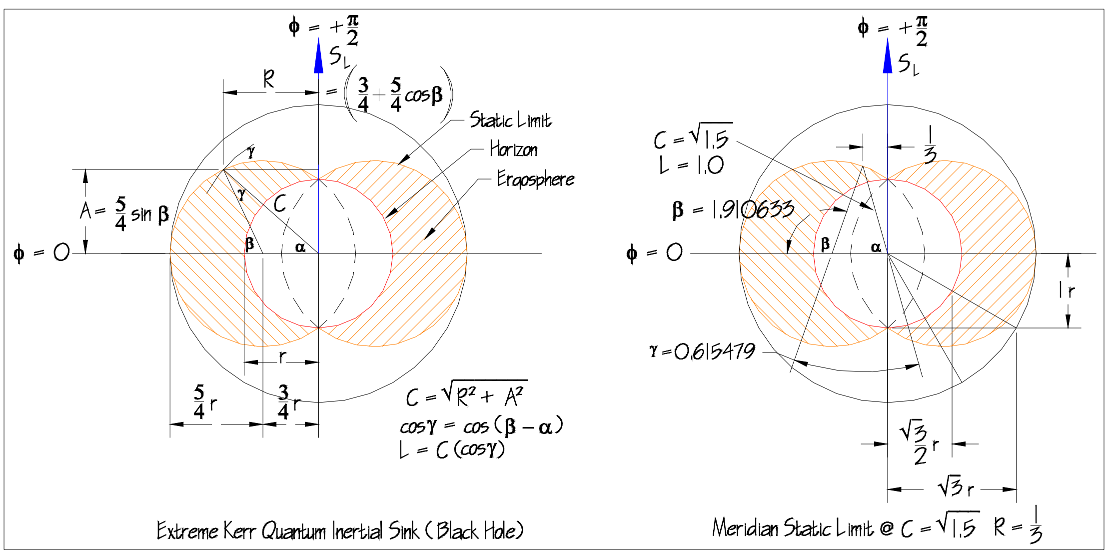
$$\pm\{a\} = \frac{\cos\left(\omega_0 t - \theta\right)}{\left|\cos\left(\omega_0 t - \theta\right)\right|} a, \quad \mp\{a\} = -\frac{\cos\left(\omega_0 t - \theta\right)}{\left|\cos\left(\omega_0 t - \theta\right)\right|} a \tag{5.11}$$

Obviously, θ and ϕ rotate at the same frequency, with the axis of the ϕ rotation rotating in the equatorial plane. Such rotation at the horizon is at the speed of light in vacuo for each rotation. This motion avoids the orientation entanglement condition as depicted in Gravitation by Misner, et al., and is necessitated by the assumed continuity condition of a classical spacetime continuum. When analyzed it is apparent that the motion is that of a transverse wave traveling in tight orbit around the spin axis, its amplitudes inclined toward the poles, analogous to a gravitationally bound, electromagnetic wave, and in fact constitutes the magnetic field of the quantum.

The following diagram is a cross-section through the spin axis and shows the relationship of the static limit, the ergosphere, and the horizon. The ergosphere is the domain of the strong interaction. The transverse or ϕ differential is limited in its motion toward the spin poles to the point on the static limit where $L = 1$.

The metric simplifies at the horizon as

$$d\tau^2 = -dt^2 + 4r_{0n}dtd\theta - R^2 d\theta^2 \mp \left\{\cos^2\left(\omega_0 t-\theta\right) L^2 d\phi^2\right\} \tag{5.12}$$



Quantum Inertial Sink Diagram 1

From Quantum Inertial Sink Diagram 1 we have the following coefficient component for ϕ along the meridians at the static limit

$$L = \tfrac{4}{5} r_{0n} + \tfrac{3}{5} R = \tfrac{4}{5} r_{0n} + \tfrac{3}{5}\left(\tfrac{3}{4} + \tfrac{5}{4}\cos\beta\right) r_{0n} = \left(\tfrac{5}{4} + \tfrac{3}{4}\cos\beta\right) r_{0n} \tag{5.13}$$

Substituting this in equation (5.12) simplifies at the horizon along the equatorial plane of a fixed spin axis where $\cos\beta = 1$, as

$$d\tau^2 = \left(idt - i2r_{0n}d\theta\right)^2 \mp \left\{\cos^2\left(\omega_0 t-\theta\right)\left(2r_{0n}\right)^2 d\phi^2\right\} \tag{5.14}$$

The corresponding spacelike metrics is

$$d\sigma^2 = -\left(idt - i2r_{0n}d\theta\right)^2 \pm \left\{\cos^2\left(\omega_0 t - \theta\right)\left(2r_{0n}\right)^2 d\phi^2\right\} \tag{5.15}$$

giving the fundamental symmetry

$$d\sigma^2 \equiv -d\tau^2 \tag{5.16}$$

and for the proper time and space, indicating the orthogonal nature of space and time,

$$d\sigma \equiv id\tau\,. \tag{5.17}$$

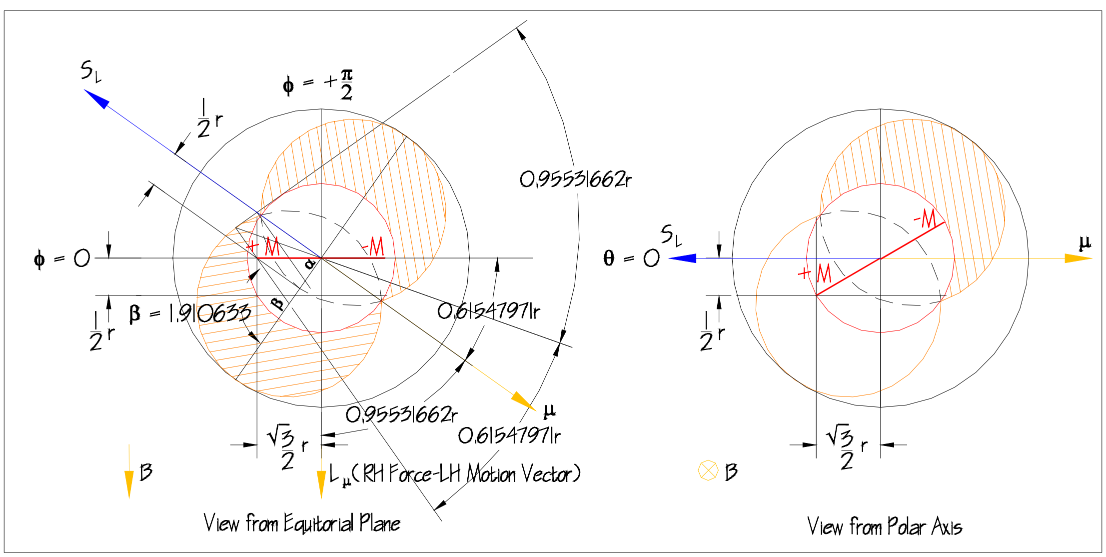
This can be represented by the following anti-symmetric orthonormal matrix at r_0,

		Direction of ortho normal vector dx_i with respect to		
		X Axis	Y Axis	Z Axis
Vector dx_i originating at	X = +1	0	$+rd\theta$	$+r\sin\omega td\phi$
	Y = +1	$-rd\theta$	0	$-r\cos\omega td\phi$
	Z = +1	$-r\sin\omega td\phi$	$+r\cos\omega td\phi$	0
	X = -1	0	$-rd\theta$	$-r\sin\omega td\phi$
	Y = -1	$+rd\theta$	0	$+r\cos\omega td\phi$
	Z = -1	$+r\sin\omega td\phi$	$-r\cos\omega td\phi$	0

Quantum Anti-Symmetric Orthonormal Matrix at r_0

In the presence of an anti-parallel external magnetic field as shown in Quantum Inertial Spin Diagram 2, the quantum spin axis inclines toward the equatorial plane and precesses about its initial position. The resulting coefficients of ½ isospin can be seen here. Note also that the Heisenberg "observational" uncertainty is limited by the inverse curvature of the horizon to

$$r_0^2 c = m_{l0} r_0 c = \hbar\,. \tag{5.18}$$



Quantum Inertial Sink Diagram 2

Conclusion

This analysis provides a physical, i.e. geometric, as well as mathematical, model of quantization, by way of a fundamental discrete oscillation/rotation, of a classical spacetime continuum that is a function of the exponential expansion of that spacetime. Quantum gravity arises naturally as the differential of that oscillatory transverse wave force with respect to expansion stress and the strong interaction as the operation of that wave force between two or more quanta within a shared force domain. This quantum state is expressed as a modification of a chargeless extreme Kerr metric with an oscillation of the ϕ coordinates imposed by continuity conditions which prevent coordinate entanglement. It thereby constitutes a physical spinor, constituting the quantum magnetic field and the property of ½ spin and isospin in the presence of other quanta. The ergosphere of this quantum metric is the domain of the strong interaction. Finally, it shows that from a universal bookkeeper reference frame, the fundamental quantum scale is the neutron scale, for which the Planck scale is the current differential.

General relativity requires the following refinement in this model. Spacetime acquires the property of inertial density as a potential energy density independent of any energy or rest mass quanta. It has an exponential expansion rate that is coupled with a covariant speed of light. It admits torsion on a quantum scale that prevents the orientation entanglement condition. Finally, quark phenomenology is shown to be the property of the nodes and antinodes of the quantum waveform.

Post Script

This model can also be developed and presented as the 3-D representation of a classical 4-D oscillation. So developed elsewhere, expansion acts as an EMF, both by mechanical analogy and actually, that drives the fundamental frequency. The rest-mass quantum is thus a small simple harmonic oscillator, with a potential-kinetic, capacitive-inductive energy cycle, in a general inductive mode during expansion, of which ordinary matter is the result. During universal contraction, a capacitive mode ensues, resulting in a predominance of anti-matter.

Over a short period of time, expansion leads to a drop in mechanical impedance, resulting in a transmission of energy and power at any neutral or resonant quantum not in nuclear congregation. The result is beta decay, which is tuned to the expansion rate for any isolated neutral quantum, and generates the electromagnetic interaction. The rest-mass ratios between the neutron, electron and proton and the “missing” mass of beta decay arises naturally in this analysis.

Endnotes

i A derivative taken on a flat rectilinear area,

$$\frac{\pm d\mathrm{A}}{dr}=\frac{\left(\mathrm{A}\pm d\mathrm{A}\right)-\mathrm{A}}{dr}=\frac{\left(r\pm dr\right)^2-r^2}{dr}=\frac{\pm 2rdr+dr^2}{dr}$$

gives a differential area of

$$\pm d\mathrm{A}=\pm 2rdr+dr^2 .$$

Now consider a hyperbolic surface, specifically the derivative of the inverse curvature of a pseudosphere, which is of constant negative curvature, for simplicity $k=-1$, where r_i is the interior radius and r_e is the exterior radius, and we have the function, where either r_e or r_i, could be used as the variable

$$k_{r_e}^{-1}=k^{-1}\left(r_e\right)=-r_e^{-1}r_e=-r_ir_e$$

The curvature is conserved, therefore the differential is zero or

$$dk_{r_e}^{-1}=\left(-r_i-dr_i\right)\left(r_e-dr_e\right)-\left(-r_ir_e\right)=r_idr_e-r_edr_i+dr_idr_e=0$$

The senses of the radii and their differentials indicate a direction toward (+) or away from (-) the exterior of the pseudosphere. Note that the differentials are of the same sense. Thus the above equation indicates a change toward the mouth or rim of the pseudosphere, as r_i is increasing and r_e is decreasing. At the point of normalization, where $r_i=r_e=r_0=1$,

we have

$$dr_e-dr_i=-dr_idr_e .$$

Therefore $dr_i=-x^{-1}, dr_e=-x \therefore dr_idr_e=1$ and after a sense inversion we have the solution

$$x-x^{-1}=1$$

$$x=\sqrt{\tfrac{5}{4}}+\tfrac{1}{2}=1.618033...=\Phi$$

the well known coefficient of conservative evolution of a system.

Note that the product $\left(x\right)\left(x^{-1}\right)1=1$ is conserved.

At the point where $r_i=\Phi^{-1}$ and $r_e=\Phi$,

we have, where the differential senses are explicit,

$$-r_i\left(-dr_e\right)+r_e\left(-dr_i\right)=-\left(-dr_i\right)\left(-dr_e\right)$$

and we can normalize the differentials at $k_{r_e}^{-1}\left(\Phi\right)$ as

$$\left|dr_i\right|=\left|dr_e\right|=\left|dr_o\right|$$

giving

$$\frac{dr_i}{dr_e}=\frac{dr_e}{dr_i}=1 ,$$

therefore

$$dr_i^2=dr_e^2=dr_0^2=d\mathrm{A}_0 .$$

Then the invariant inverse curvature is equal to the square of normalized differentials

$$k_{r_e}^{-1}\left(\Phi\right)=-r_ir_e=-dr_idr_e=-dr_0^2=-d\mathrm{A}_0=-1 .$$

However, for <u>any</u> such conservative hyperbolic system of any invariant finite curvature, we can state the following,

$$dr_i=r_i\Phi^{-1},\ dr_e=r_e\Phi ,$$

so that

$$dr_i dr_e = \left| r_i r_e \right|$$

and we have the following relationship between the inverse curvature function and its differential components

$$k_{r_e}^{-1} = k_{r_e}^{-1} + dk_{r_e}^{-1} = r_i dr_e - r_e dr_i = -r_i r_e \Phi + r_e r_i \Phi^{-1} = -dr_i dr_e = -r_i r_e \,.$$

Finally, with some substitution, for the function and its derivative, as

$$dr_i = \frac{r_i}{\Phi^2 r_e} dr_e \text{ and } r_i = \frac{-k_{r_e}^{-1}}{r_e}$$

we have, with rearrangement and simplification

$$k_{r_e}^{-1} + dk_{r_e}^{-1} = \frac{-k_{r_e}^{-1}}{r_e} dr_e - \frac{-k_{r_e}^{-1}}{\Phi^2 r_e} dr_e = -\left(1 - \Phi^{-2}\right) k_{r_e}^{-1} d\, \boldsymbol{ln}\, r_e = -\Phi^{-1} k_{r_e}^{-1} d\, \boldsymbol{ln}\, r_e \,.$$

The symmetrical condition for $k_{r_i}^{-1}$ is

$$k_{r_i}^{-1} + dk_{r_i}^{-1} = \frac{-k_{r_i}^{-1}\Phi^2}{r_i} dr_i - \frac{-k_{r_i}^{-1}}{r_i} dr_i = -\left(\Phi^2 - 1\right) k_{r_i}^{-1} d\, \boldsymbol{ln}\, r_i = -\Phi k_{r_i}^{-1} d\, \boldsymbol{ln}\, r_i \,,$$

and obviously

$$d\, \boldsymbol{ln}\, r_i = \Phi^{-1}\boldsymbol{,}\; d\, \boldsymbol{ln}\, r_e = \Phi \,.$$

Since

$$-dr_i dr_e = -\Phi^{-1} k_{r_e}^{-1} d\, \boldsymbol{ln}\, r_e = -\Phi k_{r_i}^{-1} d\, \boldsymbol{ln}\, r_i$$

we have

$$dk_{r_i r_e}^{-1} = \Phi^{-1} k_{r_e}^{-1} d\, \boldsymbol{ln}\, r_e - \Phi k_{r_i}^{-1} d\, \boldsymbol{ln}\, r_i = 0 \,,$$

and finally

$$k^{-1} = d\mathrm{A}_0 = -dr_0^2 = -dr_i dr_e = -\Phi^{-1} k_{r_e}^{-1} d\, \boldsymbol{ln}\, r_e = -\Phi k_{r_i}^{-1} d\, \boldsymbol{ln}\, r_i \,.$$

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