A Dimensional Analysis of the Dimensionless Fine Structure Constant

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By Martin Gibson

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Martin Gibson
P.O. Box 2358
Southern Pines, NC 28388
910-585-1234
martin@uniservent.org

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We can remove some of the mystery and the mysticism surrounding the fine structure constant, α, by employing the following dimensional and functional analysis. Clearly, the dimensionless context of α is due to the fact that it is a ratio of two measurements/computations of like dimensionality. It is therefore itself a factor in the coefficient of one of those dimensional structures, which we might surmise includes the dimension of time as in an impulse or force.

Using the CODATA definition of the fine structure constant, α, with fundamental or quantum charge, e_0, the reduced Planck's constant of action, $\hbar$, light speed in vacuo, c, and the permittivity of the vacuum, ε_0, we have,

$$\alpha = \frac{e_0^{\;2}}{\hbar c 4\pi\varepsilon_0} = \frac{c e_0^{\;2}\mu_0}{\hbar 4\pi}, \tag{1.1}$$

where the permeability constant, μ_0 is expressed (with dimensions shown in curly brackets) as

$$\mu_0 = \frac{1}{c^2\varepsilon_0} = q\left\{\frac{\mathrm{N}}{\mathrm{A}^2}\right\}. \tag{1.2}$$

μ_0 is defined both quantitatively, as a number q, and qualitatively (dimensionally) in terms of the magnetic force in newton$\{\mathrm{N}\}$ induced by 2 moving charges or electrical currents in ampere$\{\mathrm{A}\}$. We note that the ampere is one of the fundamental units of the International System of Units (SI) along with the three units of mass, length and time, i.e. kilogram$\{\mathrm{kg}\}$, meter$\{\mathrm{m}\}$, and second$\{\mathrm{s}\}$, which define the Newton as

$$\mathrm{N} = m \cdot l / t^2 \left\{\mathrm{kg}\cdot\mathrm{m}/\mathrm{s}^2\right\}. \tag{1.3}$$

Thus in dimensional terms, μ_0 is the ratio, here equal to q, of a force to the square of a unit current, (technically the cross product of the field strength of one unit current operating on a parallel unit current at a distance of one unit of length.) Here the naught subscripts indicate a normalized or unit value of the respective variables, or as in the case of μ_0 or ε_0, a universal constant.

Specifically, for two parallel wires, a and b, of indefinite length, one meter apart, d_0, in vacuo, each carrying a unit of current, i_0, of one ampere or coulomb$\{C\}$ per second, a magnetic force, F_{ba}, of $2x10^{-7}\mathrm{N}$ is generated on one of the wires by the other for each meter length, l_0, of the two, toward each other or positive if the currents are parallel and away from each other or negative if they are anti-parallel; thus expressed,

$$\frac{F_{ba}}{l_0} = \frac{\mu_0 i_{0a} i_{0b}}{2\pi d_0} = \frac{2x10^{-7}}{1}\left\{\frac{\mathrm{N}}{\mathrm{m}}\right\}. \tag{1.4}$$

With transposition for the value of F_{ba} as stated, we have the defined value of μ_0 as,

$$\mu_0 = \frac{2\pi F_{ba}}{i_0^{\,2}} = \frac{4\pi x10^{-7}}{\left(n_{A0}\ e_0\ /t_0\right)^2}\left\{\frac{\mathrm{N}}{\left(\mathrm{C/s}\right)^2}\right\} = \frac{4\pi x10^{-7}}{1^2}\left\{\frac{\mathrm{N}}{\mathrm{A}^2}\right\} \therefore q = 4\pi x10^{-7}. \tag{1.5}$$

In fact it is the value of μ_0 that defines $i_{0a} = i_{0b} = 1\ A\ = 1\ C/s$ and $l_0 = d_0 = 1\ m$. We note that l_0 and d_0 are orthogonal to each other. Since a coulomb is deemed to be made up of a finite number, n_{A0}, of fundamental unit charges, e_0, for each ampere or

$$n_{A0} e_0 = 1\ C \tag{1.6}$$

an ampere can be written as in the divisor of the middle term of (1.5). Substituting this into (1.1), gives

$$\alpha = \frac{c e_0^{\,2} \mu_0}{\hbar 4\pi} = \frac{e_0^{\,2} 10^{-7}}{\frac{\hbar}{c} n_{A0}^{\,2} e_0^{\,2} / t_0^{\,2}}\left\{\frac{\mathrm{C^2 N}}{\frac{\mathrm{kg{\cdot}m^2}s^{-1}}{ms^{-1}}\left(\mathrm{C^2}\right)/\mathrm{s^2}}\right\} = \frac{10^{-7}}{ת\ n_{A0}^{\,2} / t_0^{\,2}}\left\{\frac{\mathrm{N}}{\mathrm{kg \cdot m/s^2}}\right\} \tag{1.7}$$

In the last term, the constants of action and the speed of light are reduced and the unit charge squared terms are canceled. As a change in the relative placement of two charges produces a force, and a force is a momentum differential, i.e. an impulse, per unit of time, a fundamental quantum or unit of charge can be viewed as a quantum of momentum or a unit impulse, potential or kinetic, i.e. static or moving. In the next to last term, the number, n_{A0}, is the number of such impulses in an ampere of current, or when combined with one of the time dimensions, an expression of the frequency of such impulses. Canceling the fundamental charges leaves a measure of force in the antecedent, and in the consequent a frequency squared times Planck's quantum of action divided by the speed of light, which resolves dimensionally to a measure of mass-length.

In the last term, we introduce the inertial constant, ת(tav), as a time independent, fundamental mass–length unit, where for any rest mass quantum, m_q,

$$m_q = \frac{\hbar}{c^2}\omega_q = \frac{\hbar}{c}\kappa_q = ת\kappa_q \quad \therefore \frac{\hbar}{c} = ת. \tag{1.8}$$

Omega and kappa are angular wave frequency and number, respectively, indicating that mass is essentially a proxy for wave number. The inertial constant derivation is the time integral of a quantum impulse, $ת = J(t)\int_0^1 dt = m_q ƛ_{C,q}$, which is invariant.

Equation (1.7) expresses the fine structure constant as a dimensionless number. It is dimensionless in the sense that a ratio of two like qualities is dimensionless, yet such dimensionless number can also be seen as a coefficient, whole or partial, in this case as a quantifier of the consequent (divisor) of such ratio. [It is the factor required, in product with the inertial constant times the square of its frequency found in two 1 ampere currents as figured above, to produce an induced force of 10^{-7} Newton.]

Some rearrangement of (1.7) gives

$$n_{A0}{}^{2} e_0{}^{2} / t_0{}^{2} \left\{\frac{\mathrm{C}^2}{\mathrm{s}^2}\right\} = \frac{e_0{}^{2} 10^{-7}}{\alpha \text{ח}} \left\{\frac{\mathrm{NC}^2}{\mathrm{kg}\cdot\mathrm{m}}\right\} = i_0{}^{2}\left\{\mathrm{A}^2\right\} \tag{1.9}$$

Here we have an expression of the current at one ampere, squared. Once again canceling the fundamental unit charges gives

$$\left(n_{A0}/t_0\right)^2 \left\{\frac{\#^2}{\mathrm{s}^2}\right\} = \frac{10^{-7}}{\alpha \text{ח}} \left\{\frac{\mathrm{N}}{\mathrm{kg{\cdot}m}}\right\} = \frac{i_0^2}{e_0{}^{2}} \left\{\frac{\mathrm{A}^2}{\text{quantum charge}^2}\right\} = i_e{}^{2} = 3.895644...x10^{37} \left\{\left(\frac{\text{charge impulses}}{\mathrm{s}}\right)^2\right\} \tag{1.10}$$

where it is apparent that the left hand term is the square of a frequency, f_{A0}^2, perhaps periodic, semi-periodic, or some other duration. This frequency, in fact angular, gives the number of instances of the inertial moment or constant, ח, a time-independent quantum of (wave) momentum and force, found in $10^{-7}/\alpha$ N, or

$$\text{ח}\left(n_{A0}/t_0\right)^2 \left\{\mathrm{kg}\cdot\mathrm{m/s}^2\right\} = \text{ח} f_{A0}^2 \left\{\mathrm{N}\right\} = \frac{10^{-7}}{\alpha}\left\{\mathrm{N}\right\} = \alpha'\left\{\mathrm{N}\right\} = 137.035999...x10^{-7}\left\{\mathrm{N}\right\} \tag{1.11}$$

It is immediately clear that the value of α is dictated by the dividend at 10^{-7}, since the presumed invariant is their quotient, α', and a change in the dividend necessitates a corresponding change in the divisor, α, the mysterious fine structure constant. In practice, 10^{-7} sets the length of a meter in terms of c in (1.4) and (1.5), so that a nominal change in the dividend would result in a nominal change in the speed of light and in the various wavelengths found in the fine structure series, and therefore in α as well. What would change the value of α' is a change in the value of a unit of time, t_0, so that a nominal lengthening of a time unit (to include more n_{A0} per second) would nominally increase the force on the right. It is the duration of the second that determines the frequency, which determines α', and given a nominal 10^{-7}, determines α.

If we take a dimensional look at (1.2) in the context of (1.4), it appears that μ_0, the permeability of the vacuum, is converting the two current flows into a force component. It is equally correct to think that each current flow constitutes a force that interacts to produce a magnetic field force between them, their cross product given by F_{ba}, which gives μ_0 the dimensions of an inverse force. If current is expressed in units of force, then charge becomes a count of momenta or impulse as the time integral of the current, and the fundamental unit of charge becomes a fundamental unit of some type of momentum as discussed above.

This discussion is facilitated by the conceptualization of rest mass quanta as local rotational oscillations of an inertially dense spatial continuum, rather than the current Standard Model view as point particles that exhibit wave properties under certain circumstances. While a complete exposition of the mechanics of such oscillation is beyond the present scope of this monograph, (it can be provided) we can get a rough clue to this oscillation by visualizing a disk with its edge oscillating transversely about a stationary center, resulting in a wave phase rotation about that center.

Such quantum oscillations are instances of simple harmonic motion which can be expected to exhibit certain fundamental wave characteristics such as transverse wave force and transverse wave momentum, represented by a fundamental or resonant angular frequency, ω_0. This is not a body force or momentum of a particle, but rather a wave force and momentum of the density field oscillation, of which there are two simultaneous and opposed instances, one for each half of the cycle. In the quantum context, using the inertial constant as a quantum of mass-length, the fundamental transverse wave force and wave momentum are represented by

$$\sqcap\omega_0^2, \text{ fundamental quantum transverse wave force} \tag{1.12}$$

$$\sqcap\omega_0, \text{ fundamental quantum transverse wave momentum} \tag{1.13}$$

which travel around with the oscillation, with force leading momentum by $\pi/2$. If we assume for the sake of argument, that the neutron represents this fundamental oscillation, then using the mass of the neutron in light of (1.8) gives an expression and an evaluation of the wave force and momentum respectively as

$$\sqcap\omega_0^2 = \frac{\hbar}{c}\omega_n^2 = m_n c\omega_n = 716{,}766.8351 \text{ N} \tag{1.14}$$

$$\sqcap\omega_0 = \frac{\hbar}{c}\omega_n = m_n c = 5.02130545...x10^{-19} \text{ kg}\cdot\text{m/s} \tag{1.15}$$

The above is a tremendous amount of force, especially for a single quantum, but it pales in comparison to the stress found by figuring the small cross sectional area upon which the stresses operate, which are on the order of 10^{37} pascals.

Note the following ratio, in which the π converts the angular frequency to a semi-periodic frequency

$$\frac{\sqcap\omega_0/\pi}{e_0} = \frac{1.59833...x10^{-19}}{1.60217...x10^{-19}} = 0.997599940... = 1 - 0.002400060... \tag{1.16}$$

where both terms are figured in units of momentum. That is, electron charge is neutron wave momentum observed as a result of beta decay.

In the above referenced wave conceptualization, the fundamental frequency is driven by cosmic expansion, and the fundamental wave force itself is a function of the expansion stress, an isotropic stress that pervades space, isotropic that is, except at the "surface" of the oscillation where the cross product effects of microscopic torsion make themselves felt as particle spin.

The mechanical impedance, Z_0 of such inertial space, (not to be confused with the SI impedance of the vacuum), is the quotient of the fundamental characteristic wave force and the speed of wave propagation, evaluated here and compared with the last term of (1.16)

$$Z_0 = \frac{\text{ח}\omega_0^2}{c} = \text{ח}\omega_0\kappa_0 = 0.002390877.... \tag{1.17}$$

Thus

$$\frac{\text{ח}\omega_0\left(1+Z_0\right)/\pi}{e_0} = \frac{\text{ח}\omega_0\left(1+\text{ח}\omega_0\kappa_0\right)/\pi}{e_0} = .999985079... = 1-.000014921..., \tag{1.18}$$

where the subtrahend of the last term represents the difference of the antecedent on the left from a theoretically precise equation with e_0. As the value of antecedent depends on a measured observation of the neutron mass, which presumably uses Newton's constant, G, for its evaluation in terms of some mass standard, and as the relative uncertainty of that constant is 10^{-4} and (1.18) is precise to approximately 10^{-5}, these results recommend pursuit of this line of reasoning.

Combining (1.18), (1.11) and (1.4) we have an expression relating fundamental charge and electromagnetic induced force in terms of the inertial constant and the resonant frequency and wavelength of the vacuum, and in light of the above uncertainty,

$$2\text{ח}\left(\frac{\pi}{\text{ח}\omega_0\left(1+\text{ח}\omega_0\kappa_0\right)}\right)^2 \simeq \frac{2\text{ח}}{e_0^2} = 2\text{ח}f_{A0}^2 = \frac{F_{a0b0}}{\alpha} = \frac{2x10^{-7}}{\alpha}N\,. \tag{1.19}$$

With respect to the Standard Model, it has always seemed incomprehensible to me that such separate and diverse ontologies as that of the electron and proton could fit together so precisely with the help of the equally incomprehensibly elusive neutrino, to produce a neutron. It's as if an archaeologist, finding one large piece of pottery with a piece of its rim missing, next found a much smaller piece nearby that filled the missing void, but for a similar sized piece still lost, then concluded that his partial reconstruction represented the first time the pieces were ever joined! Add to this that his dig found billions and billions of similarly fitting parts. If the electron and proton (and neutrino) fit together so well, it is because they started out together as a unit. In fact what is happening, is that the expansion of the cosmos leads to a drop in the inertial density of space, and a related drop in its impedance. This produces a discontinuity at the boundary, i.e. node of the oscillation, and results in the transmission of power and energy which is registered as beta decay. The results are precisely determined by geometry, accounting for the observed neutron/electron, proton/electron, and neutron/proton mass ratios. Classical linear wave analysis, taken to three and four dimensions explicates the basics of quantum dynamics, in which it is seen that the lepton and quark phenomenology of the Standard Model can be understood as the nodal/antinodal structure of a multi-dimensional bound wave system.

There is much, much more to this analysis for the sufficiently curious party. Quantum gravity is a natural outcome of its development.