# Unification 

# —:- <br> Addressing the Fundamental Problem of Theoretical Physics 

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July 15, 2015
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#### Abstract

The fundamental problem of theoretical physics for some time has been the inability to couple the gravitational interaction and the various quantum interactions in a manner that is consistent with both models. Both general relativity as the best model for the gravitational interaction and quantum field theory as the best model for the strong and electroweak interactions have a long history of proven efficacy. What is missing is the provision of a quantum mechanism for coupling rest mass and light speed quanta with spacetime in a manner that is consistent within itself and with the established models. This monograph examines the subject with an analysis of the Planck scale in the context of the formal structure of the field equation of general relativity.


## The Planck Scale - Fundamental, Differential, or Both?

Most attempts at such provision focus on a natural regime or scale at which all basic physical properties are unitary. At such scale length, $r_{0}$, time, $t_{0}$, and mass as a measure of inertia, $m_{0}$, as well as the composite kinematic and dynamic properties such as velocity, acceleration, momentum, force, action, and energy, are treated as fundamental units. We will designate such in this presentation by the subscript naught.

The Planck scale states one such regime in which the fundamental invariants of gravitational theory as Newton's gravitational constant $G$, relativity as the speed of light $c$, and quantum theory as Planck's reduced quantum of action $\hbar$, are combined to produce a fundamental length squared value as

$$
\begin{equation*}
\mathrm{A}_{\text {Planck }}=\mathrm{A}_{0}=r_{0}^{2}=\frac{G \hbar}{c^{3}} \tag{1.1}
\end{equation*}
$$

Using SI values for the three invariants, this evaluates to $2.61 \ldots \times 10^{-70}$ meter squared.
Some additional values in light of the following dimensional analysis are:

| $r_{0}$ | $1.62 \ldots \times 10^{-35}$ meters |
| :--- | :--- |
| $t_{0}$ | $5.39 \ldots \times 10^{-44}$ seconds |
| $m_{0}$ | $2.18 \ldots \times 10^{-8}$ kilograms |

General relativity relates mass to length directly according to the following rephrasing of the relationship of (1.1) as

$$
\begin{equation*}
\frac{G}{c^{2}}=\frac{c}{\hbar} \mathrm{~A}_{0}=\frac{r_{0}}{t_{0}} \frac{t_{0}}{m_{0} r_{0}^{2}} r_{0}^{2}=\frac{r_{0}}{m_{0}} \tag{1.2}
\end{equation*}
$$

Therefore, at any scale it is assumed that

$$
\begin{equation*}
n m_{0} \frac{G}{c^{2}}=n r_{0} \tag{1.3}
\end{equation*}
$$

where $n$ is the same number for mass and length.

On the other hand, quantum theory relates individual particle mass, $m_{q}$, and length, $r_{q}$, indirectly, where $r_{q}$, is equal to the reduced Compton wavelength, $\lambda_{C_{q}}$, of a particle, a statistically derived experimental value as

$$
\begin{equation*}
m_{q} \frac{c}{\hbar}=\frac{1}{r_{q}}=\frac{1}{\lambda_{C q}} \tag{1.4}
\end{equation*}
$$

Assuming that at some scale $m_{q}=m_{0}$ and $r_{q}=r_{0}$, then (1.4) is consistent with (1.3), which recommends the Planck scale as a unifying scale.

In terms of natural units, the observed invariants are expressed as

$$
\begin{align*}
c & =\frac{r_{0}}{t_{0}}  \tag{1.5}\\
\hbar & =\frac{m_{0} r_{0}^{2}}{t_{0}}  \tag{1.6}\\
G & =\frac{r_{0}^{3}}{m_{0} t_{0}^{2}} \tag{1.7}
\end{align*}
$$

The first two of these are straightforward dimensional properties, the first as a velocity and the second as an action or angular momentum. The gravitational constant requires a bit of analysis, since its dimensional composition does not conform to any well-known physical property. For this we must turn to Newton's gravitational law, which is the context in which $G$ was experimentally derived, though it is utilized in Einstein's gravitational field equation, GFE, stated here as

$$
\begin{equation*}
R_{\mu \nu}-\frac{1}{2} g_{\mu \nu} R+g_{\mu \nu} \Lambda=\frac{4 \pi G}{c^{4}} 2 T_{\mu \nu} \tag{1.8}
\end{equation*}
$$

Newton's gravitational law expresses the force of gravitational interaction, $F_{G}$, between two bodies of celestial size and separation as the product of their individual masses, $M_{a}$, and the square of their inverse distance of separation, $d$, times an experimentally derived constant of proportionality, $G$ as

$$
\begin{equation*}
F_{G}=\frac{M_{1} M_{2}}{d^{2}} G \tag{1.9}
\end{equation*}
$$

Despite the derivation of the law in application to celestial mechanics, $G$ was first determined to a reasonable degree of precision by a laboratory experiment involving a torsion balance after the manner devised by Henry Cavendish. This of itself indicates a massive particle or quantum basis of the interaction, rather than the operation of a mechanism that is essentially celestial in nature. In the value of $G$ there must be an implied force, $\tau_{0}$, of some mechanism that effects the gravitational interaction as indicated by the following analysis of (1.7) at the Planck scale

$$
\begin{equation*}
G=\frac{r_{0}^{3}}{m_{0} t_{0}^{2}}=\frac{r_{0}^{2}}{m_{0}^{2}}\left(\frac{m_{0} r_{0}}{t_{0}^{2}}\right)=\frac{r_{0}^{2}}{m_{0}^{2}} \tau_{0} \tag{1.10}
\end{equation*}
$$

Based on the values of the Planck scale, this gives the following value for a fundamental unit of force:

$$
\tau_{0} \quad 1.21 \ldots \times 10^{44} \text { Newton }
$$

General relativity assigns such interaction not to a force but to the geometry of spacetime ie. to celestial mechanics, but as it is particle mass and energy that shapes that geometry in general relativity, it leaves open the manner in which such coupling between mass/energy and spacetime is effected. The stress energy momentum tensor of the field equation provides the answer.

From (1.3) and (1.4) we can rearrange (1.9) for any bodies of aggregate mass and aggregate distance in terms of a fundamental scale to get the following

$$
\begin{equation*}
G=\frac{n_{r_{0}}^{2} r_{0}^{2}}{n_{M_{1}} n_{M_{2}} m_{0}^{2}} F_{G 0}=\frac{r_{0}^{2}}{m_{0}^{2}} F_{G 0} \tag{1.11}
\end{equation*}
$$

In the last term of (1.11) the $n$ values in the middle term have all been set to 1 . The question, then is if it makes sense that $\tau_{0}=F_{G 0}$ and is the same for both expression (1.10) derived from the Planck scale and (1.11) derived from Newton's gravitational law. Keep in mind that in the latter expression, $G$ converts the $M_{I} M_{2}$ and $d^{2}$ terms to natural units that are then multiplied by the imputed fundamental force, $F_{G 0}$ to arrive at the interaction strength.

## The Fundamental Stress Tensor

To answer this, we turn to Einstein's field equation(1.8), to the right hand term, $2 T_{\mu \nu}$, which is the stress energy momentum tensor. While tensors customarily configure the relationship of vector transformations from a common origin, in the case of a stress tensor we prefer to use a unit cube as an origin for the tensor, in the manner of a coordinate singularity. Stress is the product of a force or forces operating on the surface of such cube, or half of such cube as indicated by three, unit surface squares with a common vertex at the point $(1,1,1)$. Table 1 shows this condition. These surfaces may be deformed or strained by an eventual stress, but the initial condition assumes they are square. The coefficient of 2 represents the sum of the original $T_{\mu \nu}$ and its geometrical negative formed by the three, unit surfaces with a common vertex at $(0,0,0)$, thereby completing the unit cube. The indices for the second tensor are oriented antiparallel to those of the first tensor. In particular this configuration represents an isotropic condition.

The $\mu$ rows of the 3-dimensional spatial matrix of the complete 4-dimensional spacetime matrix, represent these surfaces by a unit normal vector at each surface. While this component is a vector for mathematical purposes, in a classical or continuum analysis such as general relativity, it is conceptually a cross-sectional area and any force operating on or across that area is understood to be distributed over the whole cross-section. As such, these rows set the scale of the unit cube, including the unit of time which is normalized to the space scales. As with the space unit areas, the time unit is a square of a time length, therefore an acceleration factor as it applies to a force or energy components of the tensor.

In Table 1, the 9 cells in the heavy outline represent the space components of the tensor. Note that there are no time derivatives in these cells, as they represent a snap shot of the unit cube in time. The matrix product for each cell is indicated by the dimensionless strain, $\varepsilon_{i j}$, in the $v$ direction with respect to the $\mu$ unit vector designated by the crosssection or

$$
\begin{equation*}
r_{j} / \mathrm{A}_{\mathrm{oli}}=\varepsilon_{i j} \tag{1.12}
\end{equation*}
$$

The unit stress/energy density, $\rho_{0}$, is effectively a stress modulus and the product of the density and the strain gives the tension or shear/torsion stress for each cell, expressed as the ratio of the stress force, $\tau_{j}$, and the cross-section, $\mathrm{A}_{0 \mid i}$. The straight bracket divider is used to separate the unit naught indicator on the left of the subscript from the index values on the right.

The time derivatives are shown in the seven cells with wave borders for $\mu=0$ and $\nu=0$. As discussed above, the $\mu$ rows are unit scales, but the $v$ vectors are differentials which are integrated to unit values with respect to the $\mu=0$ row in this representation. They could be less, but cannot be more than one, as they are normalized to the time unit and if greater than 1 would indicate superluminal speed.

The three $\mu_{0 j}$ cells are the derivatives of a force operating on the cube with respect to the time differential. The three $v_{i 0}$ cells give the derivative of the cross-section with respect to the time differential of $v=0$, as they would be expected to change with an expansion or contraction of spacetime. Cell $\mathrm{T}_{00}$ in the double wave borders gives the time scale derivative as a change in the time scale with the passage of time. Such passing of time is gauged by the change in inertial density of flat spacetime as would be the case for an inertial spacetime as a function of a cosmological constant or the Hubble rate. Without change in density, there is effectively no time. Thus, the acceleration, either expansion or contraction, of the time scale indicated by the product that is $\mathrm{T}_{00}$, is due to a change in unit density over time, $\dot{\rho}_{0}$.

Thus, an increase in the inertial density results in a positive differential force and a negative differential area, while a decrease in density produces a negative differential force and a positive differential area with respect to tension stress. This does not indicate a violation of energy conservation, since a decrease in density in one area may be offset by an increase in another. In addition, the energy of shear and torsion stress transformed to an increasing frequency of angular motion can add to the increase in density without contraction or expansion.

The $v$ columns of this 3-D matrix represent the inertial components of a force or forces operating on and across the surfaces. They are vectors comprised of the product of a unit length vector and the unit mass value, which is equal to the inertial invariant, $\Omega$, (tav), which is itself equal to h-bar over the speed of light.

If there is a net concentric flux of inertial components, ie. momentum or force, through the surface over time, the cube is a sink. If there is a net radial flux of such components
out through the surfaces over time, the cube is a source, and if there is no change over time, the cube is divergence free. Similarly, if there is a net rotation of the surfaces of the cube over time, the cube exhibits curl, whereas if there is no such circulation or rotation, it is curl free.

| $\underset{>}{\text { Displacement }}$ | $v=0$, Time, $\int d t=t_{0}$ | $\begin{aligned} & v=1, \text { Space, } \\ & \int d r_{1}=r_{\text {oll }} \end{aligned}$ | $\begin{aligned} & v=2, \text { Space, } \\ & \int d r_{2}=r_{012} \end{aligned}$ | $\begin{aligned} & v=3, \text { Space, } \\ & \int d r_{3}=r_{013} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| Cross-section, Unit Scale, A0 V | $\frac{\hbar}{c t_{00}}=m_{0} c t_{0}=\Omega$ | $\frac{\hbar}{c r_{0}} r_{011}=m_{0} r_{011}=\Omega_{1}$ | $\frac{\hbar}{c r_{0}} r_{012}=m_{0} r_{012}=\Omega_{2}$ | $\frac{\hbar}{c r_{0}} r_{013}=m_{0} r_{013}=\Omega_{3}$ |
| $\mu=0,$ <br> Time Scale $\mathrm{A}_{010}=\left\|t_{0}^{2}\right\|$ | $\mathrm{T}_{00}, \dot{\rho}_{0}$ <br> Time Acceleration $\partial t_{0}^{2} / t_{0}=\dot{t}_{0}^{2}$ | $\mathrm{T}_{01}, \dot{\tau}_{\text {ol }}$ <br> Differential Force $\frac{-m_{0}{ }_{0}^{2}}{r_{0}} / / i_{0}^{2} /=\frac{\Omega_{1}}{\dot{t}_{0}^{2}}$ | $\mathrm{T}_{02}, \dot{\tau}_{012}$ Differential Force $\frac{-m_{0}}{r_{0}} r_{01}^{2} / i_{0}^{2}=\frac{\Omega_{2}}{\dot{t}_{0}^{2}}$ | $\mathrm{T}_{03}, \dot{\tau}_{013}$ Differential Force $\frac{-m_{0}}{r_{0} r_{01}^{2}} / i_{i_{0}^{2}}=\frac{\Omega_{3}}{\dot{t}_{0}^{2}}$ |
| $\mu=1,$ <br> Space Scale $\mathrm{A}_{011}=\left\|r_{0}^{2}\right\|$ | $\mathrm{T}_{10}, \dot{\mathrm{~A}}_{\mathrm{oll}}$ Differential Unit Area $\partial \mathrm{ron}_{10}^{2} / t_{0}=\partial \mathrm{A}_{\mathrm{ol}} / t_{0}$ | $\mathrm{T}_{11}$, Tension $\rho_{0}{ }^{r} / \mathrm{A}_{011}=\frac{\tau_{1}}{\mathrm{~A}_{011}}$ | $\mathrm{T}_{12}$, Torsion/Shear $\rho_{0} r^{r} / \mathrm{A}_{\mathrm{ol\mid}}=\frac{\tau_{2}}{\mathrm{~A}_{011}}$ | $\mathrm{T}_{13}$, Torsion/Shear $\rho_{0}{ }^{r_{3} / \mathrm{A}_{011}}=\frac{\tau_{3}}{\mathrm{~A}_{011}}$ |
| $\mu=2,$ <br> Space Scale $\mathrm{A}_{012}=\left\|r_{0}^{2}\right\|$ | $\mathrm{T}_{20}, \dot{\mathrm{~A}}_{012}$ <br> Differential Unit Area $\partial \mathrm{r}_{\mathrm{On}}^{2} / t_{t_{0}}=\partial \mathrm{A}_{02} / t_{0}$ | T21, Torsion/Shear $\rho_{0}{ }^{r} / \mathrm{A}_{02}=\frac{\tau_{1}}{\mathrm{~A}_{012}}$ | $\mathrm{T}_{22}$, Tension $\rho_{0}{ }^{r_{2} / \mathrm{A}_{02}}=\frac{\tau_{2}}{\mathrm{~A}_{012}}$ | $\mathrm{T}_{23}$, Torsion/Shear $\rho_{0}{ }^{r_{3} / \mathrm{A}_{02}}=\frac{\tau_{3}}{\mathrm{~A}_{012}}$ |
| $\mu=3,$ <br> Space Scale $\mathrm{A}_{013}=\left\|r_{0}^{2}\right\|$ | $\mathrm{T}_{30}, \dot{\mathrm{~A}}_{013}$ <br> Differential Unit <br> Area $\partial \mathrm{r}_{0 \mathrm{BI}}^{2} / I_{0}=\partial \mathrm{A}_{03} / t_{0}$ | T31, Torsion/Shear $\rho_{0}{ }^{r} / \mathrm{A}_{03}=\frac{\tau_{1}}{\mathrm{~A}_{013}}$ | T32, Torsion/Shear $\rho_{0}{ }^{r^{2} / \mathrm{A}_{03}}=\frac{\tau_{2}}{\mathrm{~A}_{013}}$ | T33, Tension $\rho_{0} r_{\mathrm{r}_{013}}=\frac{\tau_{3}}{\mathrm{~A}_{013}}$ |

Table 1- $T_{\mu \nu}$, Stress - inertial density tensor
For consistence, in terms of the 4-D spacetime tensor, we can envision an extradimensional space unit normal vector representing time which is orthogonal to the three space unit vectors for each of the doublets. For representation in 3-space, this vector would have a resultant length of $\sqrt{3}$ times each space unit vector. We will make use of this in a moment when we look at this form in detail, but first we want to get to where we are going with a scalar short cut.

While stress is essentially a tensor as described above, it can be represented analytically as a scalar force over a cross-sectional area. For a unit volume, or really for some flux through that volume or across its surface in keeping with Gauss theorem, the stress can be equated to the energy density of the volume as follows

$$
\begin{equation*}
\rho=\frac{E}{r_{0}^{3}}=\frac{W}{r_{0}^{3}}=\frac{\tau r_{0}}{r_{0}^{2} r_{0}}=\frac{\tau}{r_{0}^{2}}=f \tag{1.13}
\end{equation*}
$$

Here $f$ is the tension stress per face of the cube, $\tau$ is the force operating on that surface, W is the work done by that force operating over a length $r_{0}, \mathrm{E}$ is the energy in and at the surface, and $\rho$ is the volumetric density. A change in $\rho$ is represented by the 4-D matrix cell $T_{00}$ which is equal to the sum total of all stress, tension and shear, operating on the 3D components, $f_{i j}$, or conventionally $T_{i j}$, where the $i$ and $j$ indices represent the spatial, but not the time rows and columns.

The time row, $T_{0}$, represents the rate of change in the $v$ th vector with respect to the time scale given by $\mu=0$ as a time unit vector. In keeping with the cross-sectional aspect of the space unit vectors, the time unit vector can be thought of as a clock face that marks out one unit of time. It essentially provides for a time derivative of the inertial flux vectors. In contrast, the time column, $T_{\mu 0}$, represents the arrow or extension of time, a time integral over which some change to the unit vectors represented by $\mu$ may occur. $T_{00}$, then represents a change in the time scale over time or an inherent acceleration. It therefore represents the kinetic energy of the unit cube, while $f_{i j}$, as time free, represents the elastic potential energy density of the unit as a snapshot in time both inherently, absent any stress, and given the extant stress/strain state of the system.
$T_{0 j}$ represent the force or momentum flux, essentially force differentials at the unit surface and $T_{i 0}$ represent the change in density resulting from a change in the crosssectional areas of the cube.

Taking the total derivative of $\rho$ with respect to the second to the last term of (1.13) gives

$$
\begin{align*}
& \frac{\rho}{6 \sqrt{3}}=\frac{\tau}{r_{0}^{2}}=\frac{\tau}{\mathrm{A}_{0}} \\
& \frac{d \rho}{6 \sqrt{3}}=\frac{\partial \rho}{\partial \tau} d \tau+\frac{\partial \rho}{\partial \mathrm{A}_{0}} d \mathrm{~A}_{0}  \tag{1.14}\\
& \frac{d \rho}{6 \sqrt{3}}=\frac{1}{\mathrm{~A}_{0}} d \tau-\frac{\tau}{\mathrm{A}_{0}^{2}} d \mathrm{~A}_{0} \\
& d \rho=\frac{6 \sqrt{3}}{\mathrm{~A}_{0}} d \tau-\frac{6 \sqrt{3} \tau}{\mathrm{~A}_{0}^{2}} d \mathrm{~A}_{0}
\end{align*}
$$

The $6 \sqrt{3}$ figure arises from the above comments as a product of the trace of the doublet tensor and the orthogonal condition of $T_{00}$.

If energy is to be conserved in the unit, the net change in the density, measured by a variable, yet still unitary volume and surface, will be unchanged, even by an expansion or contraction of the unit volume. This may involve a redirection of radial to rotational strain, from the symmetric to the anti-symmetric components of the matrix. In this context we can state that the two differential components of the last line of (1.14) are necessarily equal. We can separate the two terms as follows

$$
\begin{gather*}
d \tau=\frac{\mathrm{A}_{0}}{6 \sqrt{3}} d \rho  \tag{1.15}\\
d \mathrm{~A}_{0}=\frac{\mathrm{A}_{0}^{2}}{6 \sqrt{3} \tau} d \rho=\frac{\mathrm{A}_{0}}{\rho} d \rho \tag{1.16}
\end{gather*}
$$

If we are considering this with respect to a fundamental scale, such as the Planck, then the rest of the terms in (1.13) will be unit terms, which we will restate as

$$
\begin{equation*}
\rho_{0}=\frac{E_{0}}{r_{0}^{3}}=\frac{W_{0}}{r_{0}^{3}}=\frac{\tau_{0} r_{0}}{r_{0}^{2} r_{0}}=\frac{\tau_{0}}{r_{0}^{2}}=f_{0} \tag{1.17}
\end{equation*}
$$

Doing the same for (1.15) and (1.16) and combining the two gives the invariance of the system as

$$
\begin{align*}
& d \mathrm{~A}_{0}=\frac{d \tau_{0}}{d \rho} \frac{\mathrm{~A}_{0}}{\tau_{0}} d \rho  \tag{1.18}\\
& \frac{\tau_{0}}{\mathrm{~A}_{0}}=\frac{d \tau_{0}}{d \mathrm{~A}_{0}}=\frac{\rho}{6 \sqrt{3}}
\end{align*}
$$

This of course does not mean that the differentials and the unit values are equal and in fact, $\tau_{0} \neq d \tau_{0}$ and $\mathrm{A}_{0} \neq d \mathrm{~A}_{0}$. What it means is that a unit volume change as a proportion of the total field will result in a proportional change of energy content and an invariant inertial density. Such density can still vary regionally, but the length/time scales will vary with it.

With respect to the value of $d \rho$ in (1.15) and (1.16), as with any derivative in which the value is known, it is equivalent to 1 upon such evaluation, and is required on the right hand side in the following to maintain dimensional consistency of the dependent variable differential. These two equations thus become

$$
\begin{gather*}
d \tau_{0}=\frac{\mathrm{A}_{0}}{6 \sqrt{3}} d \rho_{0}  \tag{1.19}\\
d \mathrm{~A}_{0}=\frac{\mathrm{A}_{0}^{2}}{6 \sqrt{3} \tau_{0}} d \rho_{0}=\frac{\mathrm{A}_{0}}{\rho_{0}} d \rho_{0} \tag{1.20}
\end{gather*}
$$

Clearly, the use of the valuation for $\tau_{0}$ at (1.10) in conjunction with the natural mass and length values of the Planck scale should equal the observed value of $G$. This is a tremendous amount of force and an even greater amount of stress based on a cross section of the Planck area, perhaps viable at an early cosmic epoch or in an inertial source. Let's examine another option.

Let us assume that the force given above and attributed to a quantum of gravity is a differential force and that the Planck area is a differential cross-sectional area in keeping with the stress tensor analysis above.

Inserting (1.15) into the analysis of Newton's gravitational constant at (1.11) so that $F_{G 0}=d \tau_{0}$ gives

$$
\begin{equation*}
G=\frac{r_{0}^{4}}{\hbar^{2} / c^{2}} \frac{\mathrm{~A}_{0}}{6 \sqrt{3}} d \rho=\frac{r_{0}^{6}}{\hbar^{2} / c^{2}} \frac{1}{6 \sqrt{3}} d \rho \tag{1.21}
\end{equation*}
$$

where the differential density is a dimensional placeholder equal to 1 . Rearrangement gives

$$
\begin{equation*}
r_{0}^{6}=6 \sqrt{3} G \frac{\hbar^{2}}{c^{2}} \frac{1}{d \rho}=8.58 \ldots x 10^{-95} \text { meters }^{6} \tag{1.22}
\end{equation*}
$$

so that $r_{0}=2.100 \ldots x 10^{-16}$ meters. This is the reduced Compton wavelength of the neutron, $\lambda_{C n}$.

To check ourselves, we can use the Codata value for $\lambda_{C_{n}}$ and other invariant values in (1.21) and we get $6.673198 \ldots \times 10^{-11}$. The Codata value is $6.67408(31) \times 10^{-11}$.

The gravitational quantum differential is then

$$
\begin{equation*}
d \tau_{0}=\frac{\mathrm{A}_{0}}{6 \sqrt{3}} d \rho=4.244 \ldots x 10^{-33} \text { Newton } \tag{1.23}
\end{equation*}
$$

The fundamental force, the strong force, at this scale rather than that given for the Planck scale is determined by the neutron frequency per the following

$$
\begin{align*}
& m_{n} c^{2}=\frac{\hbar}{c \lambda_{C n}} c^{2}=\hbar \omega_{n}  \tag{1.24}\\
& \tau_{0}=\tau_{n}=m_{n} c \omega_{n}=7.167 \ldots x 10^{5} \text { Newton }
\end{align*}
$$

The ratio of the strong force and the gravitational differential is then

$$
\begin{equation*}
\frac{\tau_{0}}{d \tau_{0}}=1.688 \ldots x 10^{38} \tag{1.25}
\end{equation*}
$$

This, coupled with the invariance requirements of energy density as stated in (1.18) indicates that the same relationship will hold for the cross-sectional differential, $d \mathrm{~A}_{0}$, given the following, where the fundamental scale area is given by the square of the reduced neutron Compton wavelength as

$$
\begin{equation*}
\mathrm{A}_{0}=r_{0}^{2}=\lambda_{c, n}^{2}=4.410 \ldots x 10^{-32} \text { meter }^{2} \tag{1.26}
\end{equation*}
$$

Thus we have

$$
\begin{equation*}
d \mathrm{~A}_{0}=\frac{d \tau_{0}}{\tau_{0}} \mathrm{~A}_{0}=2.61 \ldots x 10^{-70} \text { meter }^{2} \tag{1.27}
\end{equation*}
$$

and we see that the Planck scale is a differential scale of the spacetime continuum and not a discrete scale.

The fundamental stress is therefore

$$
\begin{equation*}
\frac{\tau_{0}}{\mathrm{~A}_{0}}=1.625 \ldots \times 10^{37} \text { Newton } / \text { meter }^{2} \tag{1.28}
\end{equation*}
$$

and the inertial density is

$$
\begin{equation*}
\rho=\frac{E}{V}=1.688 \ldots x 10^{38} \text { Joules } / \text { meter }^{3} \tag{1.29}
\end{equation*}
$$

This analysis demonstrates that the unification scale for gravity and the other quantum interactions is the neutron scale.

Returning to an analysis of the spacetime stress tensor as developed above, there are essentially three conditions in addition to an initial condition in which stress and strain are absent, therefore absent change and thus time. With respect to the Einstein field equation, all the remaining terms on the right-hand side of the equation are redundant, unitary, or normalized in light of the above analysis. The $2 T_{\mu \nu}$ term includes the surface integral represented by the $4 \pi$ term, $G$ as a gravitational quantum is included as the differential components of the force, and the length and time scales of the speed of light quad are normalized in the four rows and columns of the matrix.

1. Of the three, the first condition is the vacuum state in the absence of transiting messenger or resident rest mass phenomenon. Here, assuming a positive cosmological constant, $\Lambda$, as indicated by the Hubble rate, $H_{0}$, the unit of spacetime is either expanding or contracting, depending on whether we consider the cosmic manifold to be expanding or fixed over time. In either case, the time row and column values are non-trivial, but the resulting field is flat over the near term, that is it transmits the gravitational force along the normal unit vectors according to the symmetric portion of the tensor, and while we can anticipate fluctuations in the shear components, it is essentially curl free.
2. The second condition is that of a transiting messenger, but no rest mass, phenomenon. In addition to the dilatation of the vacuum state, the tensor will exhibit torsion and directional flux with the transit, but no sustained divergence or curl.
3. The third condition is that of the generation of fundamental rest mass. In this state, the vacuum state has progressed to the point at which transverse or shear strain components of the stress/strain relationship result in orthogonal torsion oscillation which in turn results in rotation of the field stress in the antisymmetric components. Note that it is a rotation of the field stress phases and not a physical rotation, though there is an oscillation and alternation of strain curl normal to the direction of rotation.

As a phase change between moments of relative maximum displacement and momentum, maximum potential and kinetic energy, this oscillation is responsible for the generation of quantum charge, which is dimensionally a measure of fundamental momentum and which initially remains confined to the unit oscillation. This strain oscillation and stress rotation produces fundamental rest mass particle spin and the magnetic dipole moment. As an instance of confined oscillation, the waveform has a sustained nodal-anti-nodal structure and sustained moments of maximum power and action that rotate with the stress phases. This
structure is disturbed over brief time frames by energetic particle collision, producing the perceived quark nature of this structure in accelerator experimental results.

Continued expansion according to $\Lambda=H_{0}$ produces in a drop in mechanical impedance as an ambient condition of $2 T_{\mu \nu}$ which results in a transmission of a small portion of the oscillation energy and power to produce the electron oscillation. The strain state in the fundamental oscillation results in a rotational axis flip and frequency reduction to produce the quantum state of the proton. As a rest mass oscillation, the emitted electron is confined to a sub-light speed trajectory, however the counter-centripetal stress responsible for the emission comprises a signature stress transmission that travels at the speed of light as the neutrino.

While this covers the basic analysis of the right hand side of the GFE (1.8), we need to examine the left hand side. For our purposes here, we are primarily concerned with the case of a single fundamental unit of spacetime, therefore with a flat spacetime, other than the strain produced at the quantum tensor. From a cosmological perspective, (1) is an initial condition of cosmic expansion, from which (3) arises. With continued expansion, and beta decay in (3), (2) arises. We would also state that the condition in the vast voids of space between galactic filaments is primarily a combination of states (1) and (2), perhaps primarily (2), with infrequent instances of (3).

In the case of flat spacetime, the Ricci components which form the Einstein curvature tensor become trivial, leaving only the product, $g_{\mu \nu} \Lambda$, on the left. It is understood, however, that in the galactic environment, and in particular in the regions of galactic black holes, the curvature tensor is hardly trivial. In fact, it is my belief that such black holes, in particular active galactic nuclei, are inertial sources rather than inertial sinks, though some may function in both capacities and that the conditions necessary for the generation of state (3) are found primarily at the surface of such non-singularity black holes, and not in a universal big bang. The expansion of spacetime as indicated by the Hubble rate, which in the full development of this model is an exponential rate, drives this generation of (3).

## The Fundamental Quantum Metric

We turn now to the metric, specifically a chargeless extreme Kerr metric in the equatorial plane (the $\phi$ coordinates are suppressed), in which the angular momentum parameter, $a$, is equal to the horizon reduced circumference and the geometrized mass, or $a=r_{h}=M_{l}$.
(Note in the discussion of this section that $\tau$ indicates the proper time and not the fundamental force of the prior section, $\tau_{0}$.) The time-like metric at the horizon is

$$
\begin{equation*}
d \tau^{2}=\left(1-\frac{2 M_{l}}{r_{h}}\right) d t^{2}+\frac{4 M_{l} a}{r_{h}} d t d \theta-\frac{d r^{2}}{\left(1-\frac{2 M_{l}}{r_{h}}+\frac{a^{2}}{r_{h}^{2}}\right)}-\left(1+\frac{a^{2}}{r_{h}^{2}}+\frac{2 M_{l} a^{2}}{r_{h}^{3}}\right) r_{h}^{2} d \theta^{2} \tag{1.30}
\end{equation*}
$$

Substituting for $a=M_{l}$ gives

$$
\begin{equation*}
d \tau^{2}=\left(1-\frac{2 M_{l}}{r_{h}}\right) d t^{2}+\frac{4 M_{l}^{2}}{r_{h}} d t d \theta-\frac{d r^{2}}{\left(1-\frac{M_{l}}{r_{h}}\right)^{2}}-\left(r_{h}^{2}+M_{l}^{2}+\frac{2 M_{l}^{3}}{r_{h}}\right) d \theta^{2} \tag{1.31}
\end{equation*}
$$

We make the following observation concerning the $d r^{2}$ term. While the conventional interpretation is that the term goes to infinity as the denominator approaches zero, and any infalling test particle transits the horizon, the math can also be interpreted in terms of a limit for radial motion. A mathematical conflation is at work in the formulation, since the differentials are deemed to approach zero in the limit, but are effectively treated as dimensional units, ie. equal to one of some infinitesimal scale. This is necessary since the product of a non-zero co-efficient and a zero differential at the limit would be zero. This is warranted since we find a similar non-zero differential without a coefficient on the left side of the equation.

This is contradicted, however, if the metric component represented by the differential has a natural limit where it is necessarily zero. Thus, if the horizon in an extreme Kerr spacetime represents that limit, $d r$ equals zero at the limit of that horizon coincident with the term in the denominator, the coefficient and the differential cancel. The result is simply -1 as shown below, which when factored gives an imaginary or orthogonal sense, ie. it rotates any differential change into tangency. The horizon, then, is effectively a physical asymptote. Thus, at the event horizon, where $r=r_{h}=M_{l}$ this simplifies to

$$
\begin{equation*}
d \tau^{2}=-d t^{2}+4 r_{h} d t d \theta-(2 r)^{2} d \theta^{2}-d r^{2}=\left(i d t-i 2 r_{h} d \theta\right)^{2}+(i d r)^{2} \tag{1.32}
\end{equation*}
$$

This can be factored as a complex number and its conjugate

$$
\begin{equation*}
d \tau^{2}=\left[\left(i d t-i 2 r_{h} d \theta\right)+i(i d r)\right]\left[\left(i d t-i 2 r_{h} d \theta\right)-i(i d r)\right] \tag{1.33}
\end{equation*}
$$

or can be simplified as follows,

$$
\begin{equation*}
d \tau^{2}=\left[\left(i d t-i 2 r_{h} d \theta\right)-d r_{h}\right]\left[\left(i d t-i 2 r_{h} d \theta\right)+d r_{h}\right] \tag{1.34}
\end{equation*}
$$

where $r_{h}$ is the reduced circumference at the horizon and $d r_{h}=0$ is a zero vector with respect to the radial, giving a proper time of

$$
\begin{equation*}
d \tau= \pm i\left(d t-2 r_{h} d \theta\right) \tag{1.35}
\end{equation*}
$$

If we assume that for bookkeeper time the differential is in the plane of the horizon, and time as developed earlier flows with the rotational motion of the ergo-sphere, so that

$$
\begin{equation*}
d t=r_{h} d \theta \tag{1.36}
\end{equation*}
$$

then the proper time is found to flow orthogonally to that rotational motion, into the negative and positive $\phi$ coordinates, since

$$
\begin{equation*}
d \tau=\mp i d t \tag{1.37}
\end{equation*}
$$

This will be significant in our statement of the quantum metric.
From this perspective, at the static limit and the start of the ergo-sphere, where $r=2 M_{l}$, pure radial motion is no longer possible, and a rotational component or frame dragging element is injected into the equation so that at the event horizon, all motion is rotational
as indicated by the "imaginary" or orthogonal senses. Note that if we consider spacetime as an inertio-elastic continuum, frame dragging is simply the wave strain associated with a rotational waveform, be it macrocosmic or quantum. Instead of gravitational collapse, this argues that any incremental matter or light accruing to the inertial sink is smeared out and bound at the horizon.

We now get to the meat of the matter with an expression of the quantum metric, $q_{\mu \nu}$, which we distinguish from the gravitational metric, $g_{\mu v}$. The dynamics of the quantum waveform is not extremely complicated, but it does involve some rather lengthy, nonstandard analysis using methods of complex classical wave physics extended to 4 dimensions and is beyond the scope of the present discussion. We will simply state that its dynamics prevent the orientation entanglement condition.

With reference to Quantum Inertial Sink Diagram 1, the time-like quantum metric is given as a modified chargeless extreme Kerr metric. The modification is in the $\phi$ coordinates as shown here, where the quantum mass has been explicitly geometrized as

$$
d \tau^{2}=\left(1-\frac{2 r_{0 n}}{r_{0 n}}\right) d t^{2}+\frac{4 r_{0 n}^{2}}{r_{0 n}} d t d \theta-\frac{d r^{2}}{\left(1-\frac{r_{0 n}}{r_{0 n}}\right)^{2}}-R^{2} d \theta^{2} \mp\left\{\left(e^{ \pm i\left(\omega_{0} \tau \mp \theta\right)} L d \phi\right)^{2}\right\}
$$

The caveat stated above concerning the limit of radial motion represented by $r_{0 n}$ remains. In the last term, the complex exponential is defined as

$$
\begin{align*}
e^{ \pm i\left(\omega_{0} t \neq \theta\right)}=\operatorname{Re}\left(e^{ \pm i\left(\omega_{0} t+\theta\right)}\right) & =\cos _{c c w}\left(\omega_{0} t+\theta\right) \text { or } \cos _{c w}\left(\omega_{0} t+\theta\right)  \tag{1.39}\\
& =\cos \left(\omega_{0}(+t)-\theta\right) \text { or } \cos \left(\omega_{0}(-t)+\theta\right)
\end{align*}
$$

Either the real or the imaginary part could of course be used. The $c c w$ term indicates rotation in the upper hemisphere according to the right-hand rule, while the $c w$ term indicates clockwise rotation in the bottom hemisphere according to the left hand rule, when viewed from the exterior of the corresponding rotational pole.

The plus and minus curly bracket has the following definition and indicates a flipping of the sign of the $d \phi \quad$ vector, with every $\pi$ rotation of $\theta$, plus being parallel and minus being anti-parallel with respect to the RHR spin axial vector. It thus performs a function similar to a mathematical spin matrix.

$$
\begin{equation*}
\pm\{a\} \equiv \frac{\cos \left(\omega_{0} t-\theta\right)}{\left|\cos \left(\omega_{0} t-\theta\right)\right|} a, \quad \mp\{a\} \equiv-\frac{\cos \left(\omega_{0} t-\theta\right)}{\left|\cos \left(\omega_{0} t-\theta\right)\right|} a \tag{1.40}
\end{equation*}
$$

Obviously, $\theta$ and $\phi$ rotate at the same frequency, with the axis of the $\phi$ rotation rotating in the equatorial plane. This motion avoids the orientation entanglement condition and is necessitated by the assumed continuity condition of a classical spacetime continuum and the density property postulated in this development. When analyzed it is apparent that the motion is that of a transverse wave traveling in tight orbit around the spin axis, its
amplitudes inclined toward the poles, analogous to a gravitationally bound, electromagnetic wave, and in fact constitutes the magnetic field of the quantum.

This diagram is a cross-section through the spin axis and shows the relationship of the static limit, the ergo-sphere, and the horizon. The ergo-sphere is the domain of the strong interaction. The transverse or $\phi$ differential is limited in its motion toward the spin poles to the point on the static limit where $L=1$.

The metric simplifies at the horizon with no radial motion as

$$
\begin{equation*}
d \tau^{2}=-d t^{2}+4 r_{0 n} d t d \theta-R^{2} d \theta^{2} \mp\left\{\cos ^{2}\left(\omega_{0} t-\theta\right) L^{2} d \phi^{2}\right\} \tag{1.41}
\end{equation*}
$$



## Quantum Inertial Sink 1

From this diagram we have the following coefficient component for $\phi$ along the meridians at the static limit

$$
\begin{equation*}
L=\frac{4}{5} r_{0 n}+\frac{3}{5} R=\frac{4}{5} r_{0 n}+\frac{3}{5}\left(\frac{3}{4}+\frac{5}{4} \cos \beta\right) r_{0 n}=\left(\frac{5}{4}+\frac{3}{4} \cos \beta\right) r_{0 n} \tag{1.42}
\end{equation*}
$$

Substituting this in equation (1.41) simplifies at the horizon along the equatorial plane of a fixed spin axis where $\cos \beta=1$, as

$$
\begin{equation*}
d \tau^{2}=\left(i d t-i 2 r_{0 n} d \theta\right)^{2} \mp\left\{\cos ^{2}\left(\omega_{0} t-\theta\right)\left(2 r_{0 n}\right)^{2} d \phi^{2}\right\} \tag{1.43}
\end{equation*}
$$

The corresponding spacelike metric is

$$
\begin{equation*}
d \sigma^{2}=-\left(i d t-i 2 r_{0 n} d \theta\right)^{2} \pm\left\{\cos ^{2}\left(\omega_{0} t-\theta\right)\left(2 r_{0 n}\right)^{2} d \phi^{2}\right\} \tag{1.44}
\end{equation*}
$$

giving the fundamental symmetry

$$
\begin{equation*}
d \sigma^{2} \equiv-d \tau^{2} \tag{1.45}
\end{equation*}
$$

and for the proper time and space, indicating the orthogonal nature of space and time,

$$
\begin{equation*}
d \sigma \equiv i d \tau \tag{1.46}
\end{equation*}
$$

This can be represented by the following anti-symmetric orthonormal matrix at $r_{0}$,

|  |  | Direction of or | normal vector | with respect to |
| :---: | :---: | :---: | :---: | :---: |
|  |  | X Axis | Y Axis | Z Axis |
|  | $\mathrm{X}=+1$ | 0 | $+r d \theta$ | $+r \sin \omega t d \phi$ |
|  | $\mathrm{Y}=+1$ | $-r d \theta$ | 0 | $-r \cos \omega t d \phi$ |
| $\begin{aligned} & \stackrel{\rightharpoonup}{\sigma} \\ & 0.0 \end{aligned}$ | $\mathrm{Z}=+1$ | $-r \sin \omega t d \phi$ | $+r \cos \omega t d \phi$ | 0 |
| En | $\mathrm{X}=-1$ | 0 | $-r d \theta$ | $-r \sin \omega t d \phi$ |
| -8080 | $Y=-1$ | $+r d \theta$ | 0 | $+r \cos \omega t d \phi$ |
| $\begin{aligned} & \text { O} \\ & \text { P } \end{aligned}$ | $\mathrm{Z}=-1$ | $+r \sin \omega t d \phi$ | $-r \cos \omega t d \phi$ | 0 |

Table 3-Quantum Anti-Symmetric Orthonormal Matrix at $\boldsymbol{r}_{\mathbf{0}}$
In the presence of an anti-parallel external magnetic field as shown in Quantum Inertial Spin Diagram 2, the quantum spin axis inclines toward the equatorial plane and precesses about its initial position. The resulting coefficients of $1 / 2$ spin can be seen here. Note also that the Heisenberg "observational" uncertainty is limited by the inverse curvature of the horizon to

$$
\begin{equation*}
r_{0}^{2} c=m_{l 0} r_{0} c=\hbar \tag{1.47}
\end{equation*}
$$



Quantum Inertial Sink 2
This model is elsewhere more fully developed and presented as the 3-D representation of a classical 4-D oscillation. Expansion acts as an EMF that drives the fundamental frequency, both by mechanical analogy and as the actual mechanical or piezoelectric basis for electro-magnetism. The rest-mass quantum is thus a small simple harmonic oscillator, with a potential-kinetic, capacitive-inductive energy cycle, in a general inductive mode during expansion, of which the waveform of ordinary matter is the result. During universal contraction, a capacitive mode ensues, resulting in a predominance of anti-matter.

## The Fundamental Rest Mass Field Equation in Flat Spacetime

In flat spacetime in which the Ricci curvature tensor and scalar vanish, the product of the quantum metric, $q_{\mu \nu}$, and the cosmological constant/Hubble rate is equal to the fundamental stress tensor for a rest mass oscillation or

$$
\begin{equation*}
q_{\mu \nu} H_{0}=2 T_{\mu \nu} \tag{1.48}
\end{equation*}
$$

Here the Hubble rate gauges the expansion and the development of the time row and column of the doublet. In essence, $T_{00}=H_{0}$, and is a spacetime strain.

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